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STATISTICAL ANALYSIS IN FUNCTIONAL EVALUATION OF INSULATION.(U)

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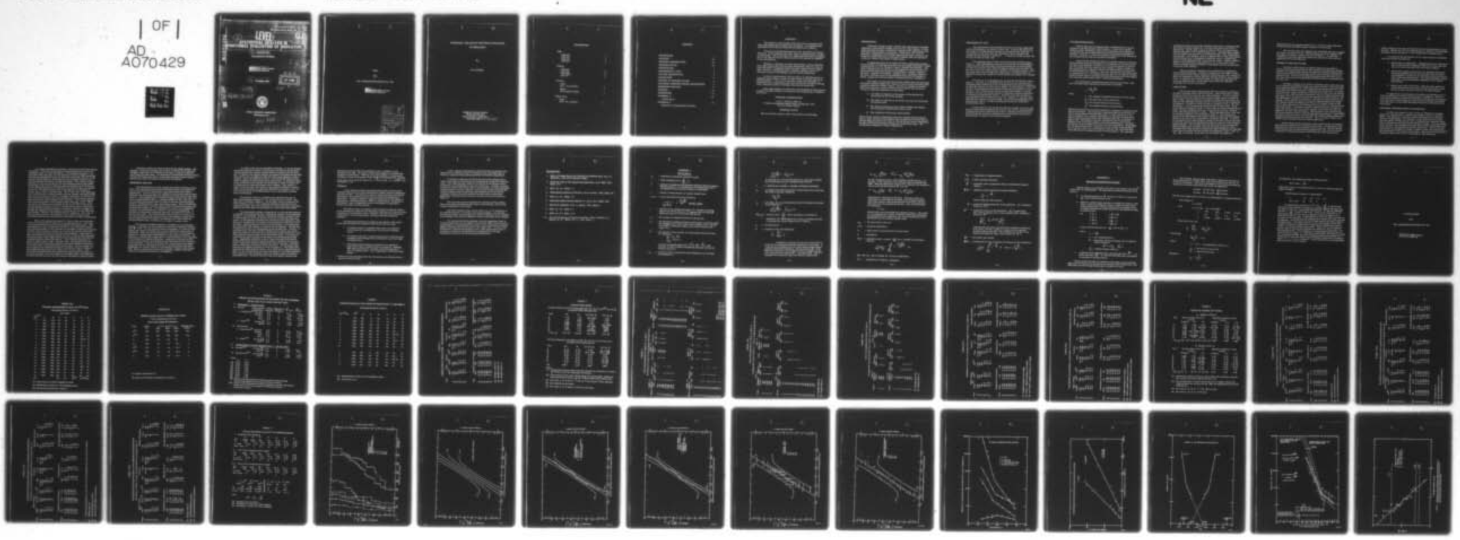

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(10) John Cybulski

ELECTRICITY DIVISION

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STATISTICAL ANALYSIS IN FUNCTIONAL EVALUATION OF INSULATION

By

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ABSTRACT

An analysis is made of data which has been accumulated from cyclic heat aging tests on magnet wire insulation. It is found that the distribution of coil failures along the test mandrel are random.

The form of the distribution function for the failure occurrence with time, of the combined test data, can be considered either normal or log-normal. This is indicative of a slight skewness which is probably due to the inherent failure characteristic of the material as supplied by the manufacturer.

Statistics in the form of means and variances can be calculated from experimental data which has been deliberately reduced in size or censored without a significant difference when compared to the original data. However, unacceptable data is obtained whenever there is a continued increase of the heating cycle for a given sample size.

A criteria approximated from the theory of rounding errors is suggested which defines a region of experimental conditions where data may be gathered. The loss of precision of data and reliability of handling in tests of significance determine the bounds of length of heating cycle, maximum number of heating cycles and the sample size.

Final representation of results by a life-temperature characteristic is shown including confidence and tolerance limits expressed on samples and curves.

PROBLEM AUTHORIZATION

N. R. L. Problem 33E07-01
Problem Assignment Request dated 15 September 1952
R. D. B. Number 480120

PROBLEM STATUS

This is an interim report; work on the problem is continuing.

INTRODUCTION

Statistical analysis of data, obtained from heat aging of insulation employed in reproducible samples such as coils, model motors, or even small operating machines, provides an estimate of the significance of the data from a given number of specimens. Conversely, for a desired probability, hypotheses can be tested regarding the mean, variance, and range of a sample and limits constructed to sample data and regression curves as a result of the design of experiments.

In aging tests there are a number of factors influencing the aging characteristics of insulating materials; they are: aging temperature, length of aging period or cycle, dielectric stress, vibration and humidity. The effect resulting from the variation in the magnitudes of these factors are best sorted out when such factors as distribution, variance, standard deviation, coefficient of variation, standard error, confidence limits on the mean, tolerance limits on the population (from which the sample is taken), sample size and the number of heating cycles are considered.

This report is concerned with the data obtained from the heat aging of Formex insulated magnet wire wound in a form of coils and tested under conditions outlined in another report[1]. Its aim is to employ the results which are in-the form of hours of life of each of the 20 coils in each of 7 tests at a different temperature to determine:

1) The effect of reduction of the number of observations (by decreasing the size of the sample),

2) The effect of reduction in the number of cycles (by increasing the heating time),

3) The effect of grouping of data in their heating cycle against the recording of instantaneous failures, and

4) The reliability of 50 percent censored data.

These various means of handling the data were studied with the ultimate user in mind, and with the assumption that the final form of these tests on the heat aging insulation will result in a plot of the life versus temperature, all other conditions being held constant at a standardized value. The nomenclature employed is listed in Appendix A.

EXPERIMENTAL DATA

The experimental data obtained for seven 20-coil heat aging tests, cycled with humidity, are shown in Table 1 (a). In two of the tests there were only 19 recorded failures because of an experimental difficulty. It is noted that in two other tests the last failures were rejected in the analysis because they were "outliers"[2]. A summary of data on each coil test may be seen in Table 1 (b).

The ungrouped data in Table 1 (a) is plotted on probability paper, Figure 1. The stepped graphs indicate the percentage of the distribution represented by each observation and also between any two observations. A curve can be drawn more easily through the points located midway[3] on the riser; the curve represents the cumulative distribution function, while the points reflect the average of the observation. Probability paper is employed because the cumulative normal distribution appears as a straight line.

First of all, an investigation was made of the distribution of these failures. Because it is desirable that the failures be stochastically independent (the probability of one not dependent on the other) the sequence in which the failures occurred were studied. For this purpose, each of the 20 coils were numbered consecutively to identify their position on a test mandrel. Coil numbering also served to determine whether or not the location of any one coil on the mandrel is subjected to unique combination of vibration and temperature effects. Analysis of the data revealed no wide variation from randomness in any test. However, it was also noted that 85% of the failures, on the average, occurred at the beginning of the heating cycle, that is, during the heating period directly following a humidification cycle.

In this analysis, one can assume that each sample of size 20 came from the same parent population for coils for any one test were generally wound from the same 5-pound spool of wire, although it would have been desirable that the sample be obtained from several spools of the same manufacturer to insure a more random sampling.

FAILURE DISTRIBUTION

Failure data may be obtained either instantaneously or may be grouped by the heating cycle characteristic. It turns out in neither instance could a quantitative check of the distribution of the failures of any one 20-coil test be obtained because of the limitations outlined by Cramer[4]. However, a check can be made to determine if the failures of the combined seven tests are normally distributed.

In any normal distribution there is a random variation of observations allowable about the theoretical curve dependent on the sample size. The theoretical allowable variation for a normal distribution of size 20 and 100 sample with zero mean and unit variance is shown in Figure 2. The approximations employed to derive the 95% confidence limits about the distribution curve, Figure 2, lose their validity when the sample size is reduced below 25. For sample size less than 25 it is difficult to differentiate between random variations about the theoretical curve and systematic deviations which would dictate a different distribution.

By combining the instantaneous data from the seven tests, with each test observation separately transformed to a deviate,

$$u = \frac{x_{ik} - \bar{x}_k}{s_k}$$

where

u = the multiple of standard deviations from the mean

$\bar{x}_k$ = the mean life in hours of kth test

x_{ik} = the ith observations of the kth test

s_k = the standard deviation of the kth test.

Sufficient observations were then available for a graphical evaluation of the failure distribution. The ungrouped transformed data from all tests, as well as the grouped transformed data with 0.2 class interval of u, are shown on Figure 3(a). Similar information was obtained for these tests with the failures grouped within their heating cycles and is presented in Figure 3(b). Both Figures 3(a) and 3(b) indicate that the combined failures of the seven tests whether recorded instantaneously or grouped by heating cycles probably come from a normal distribution. By grouping the 136 observations into 0.2 intervals of u the number of plotted points were reduced to about 20 in both Figures 3(a) and 3(b), and would have been sufficient for a graphical check of the distribution. The ungrouped

instantaneous data of each of the individual seven tests is compared with the random variation allowed for a normal distribution of 20 coils in Figure 4(a). Similar data is shown in Figure 4(b) for the data of each test when grouped in the original heating cycle. Although neither Figure 4(a) nor Figure 4(b) disprove a normal distribution, other distributions are possible. For example, Test A in Figures 4(a) or 4(b) could equally well be indicative of a rectangular distribution since it is higher than normal at both edges and lower down about the center. These variations appear to be symmetrical and systematic, but it may well be only a random variation making it difficult to distinguish from the normal when using such small samples.

There are several variables to be considered in a quantitative check of the distributions. Among the variables are the length of class intervals (which form cells), number of observations per cell, number of cells and construction of these intervals with and without a zero midpoint of (u). A summary of the results appear in Table 2 for ungrouped and cyclic grouped test data. Of the eight checks made five indicated acceptance of a normal distribution.

SAMPLE SIZE

The 20-size sample experimental data was utilized to determine the effect of sample size on the final results. Rather than take a random sample of size 10 out of the 20, the coils numbered 1 - 10 were selected from each of the seven tests and are listed in Table 3(a) as a sample of size 10 data. This procedure is considered satisfactory since there was found no interdependence between the first 10 and the second 10-numbered coil failures. In order to construct data for samples of size 5, the coils numbered 1 - 5 were selected and are also listed in Table 3(b). These seven tests consisting now of 20, 10 and 5 coils each were analyzed for the mean, variance, standard deviation, coefficient of variation, standard error, and average number of cycles. These ungrouped data, which will be called the statistics, are presented in Table 4. The simplest test which can be made between the reduced sample size data and the original sample of 20 coils is the comparison of calculated mean life. In the test of the equality of the mean (t-test) a variance ratio test (v^2) must preceed it for if the variances are significantly different, then the t-test cannot be applied directly. The variance ratio test and t-test are associated with levels of significance normally chosen at the 5% level; that is, if the calculated test values exceed the significant values tabulated, then the data occurs less than 5% of the time and the test hypothesis of $\sigma_1^2 = \sigma_2^2$ or $\xi_1 = \xi_2$ is rejected. Since the 5 and 10-sample size data was

abstracted from the original sample of 20, a frequency below 5% would indicate that the attempted sample reduction is not justified.

The variance ratio test applied to the 10-sample size and 5-sample size data, as compared to the original sample size of 20, together with the equality of means, t-values, are seen in Table 5. In both these reductions of data the difference between the respective means was not significant even at the 20% level.

NUMBER OF HEATING CYCLES

The effect of the number of heating cycles upon the ability to abstract reliable experimental data was checked by grouping the original instantaneous experimental data by the length of time of the heating cycle. Tests A and B had 100-hour heating cycles, test C through G had 4-hour heating cycles. Because of the different heating cycles and mean life of each test, there were correspondingly different numbers of heating cycles for each test.

A further variation in the number of heating cycles within each test can be obtained by grouping the data into cycles of 2 and 4 times the length of the original heating cycle. Each subsequent grouping constituted a new set of data with reduced numbers of heating cycles in each test. A similar reduction in the number of heating cycles can be carried out on the sample size of 10 data, which had been constructed from the original sample of 20. The aging cycles for the seven tests were modified in this manner for 20S and 10S samples and listed in Tables 6, 7(a) and 7(b) respectively for the original, twice original, and four times the original heating cycle length. The reduced number of heating cycle data for the sample of size 20 and size 10 were analyzed for the statistics which are shown in Tables 8(a) and 8(b) respectively. In a manner similar to that employed for sample size study, the variance ratio test and the equality of the means test may be applied to this new constructed data. Data grouped by the original heating cycle was compared with the original ungrouped observations. Observations grouped into cycles of twice and quadruple the original heating period were compared with the original cycle results. There is listed in Table 9 the v^2 and t-test calculated values for both sample sizes.

In the results of size 20 samples, tests B and G were unacceptable when data were grouped into original heating cycles. No additional t-test rejections occurred on any of the other tests of size 20. The fact that the tests B and G were not rejected when grouped into cycles twice as long is immaterial for whenever grouped observations are first rejected any

further changes in the same direction (to lower cycles) logically must be rejected. Sample size of 10 is unsatisfactory for test E at 1/4 the number of original cycles; average of 1.7 and maximum of 3 heating cycles.

It is believed that the reasons for the small rejection of modified cycles and sample size data are:

1) Small levels of significance, although these were raised from 5% to 10% and even to 20% until differences of the mean that only occur more than 20% of the time were accepted.

2) Observations within each set were not randomly drawn from the same population as such but were taken from the same sub-population of sample of size 20 which restricts the variations of the mean and variance. The reduced sample variance and mean is not stochastically independent of the original observations.

3) Sample size may be too small. When the same difference of the mean and the same variance exists the larger t-value and more likely rejections occur for the larger sample.

In Appendix B is a discussion of the effects of rounding errors to which the heating cycle failure characteristic is analogous. It will be seen that the equality of means and associated variance ratio test become more efficient for determining the minimum size of sample and heating cycle when there is one-sided censoring of the data as discussed in the next section.

ONE-SIDED CENSORED NORMAL DISTRIBUTION

To effect an economy of time it would be desirable if the aging tests were concluded after a specific percentage of the test coils had failed. The deliberate censoring of experimental data by the observer has a few disadvantages. For example, if only 50% of the coils under test have failed, the assumption must be made that the remainder would fail in conformance with the assumed statistical model. From the graphs of the test data it has already been shown that the failure distribution can most simply be represented by a normal distribution. With a censored distribution an assumption of $n \rightarrow \infty$ is made in order to be able to calculate an estimate of the variance of the mean. For the seven tests there is a saving of 30% in test time on the average for 50% censoring.

In a proposed graphical procedure, only the first half of the failures are needed. It has been found that by simply plotting these failures on probability paper in a step-like fashion, as illustrated in Figure 1, for ungrouped 20-sample data, that the intersection with the 50% fractile (the last failure) approximates the mean while the intersection with the 15.9% fractile will be indicative of the mean less the standard deviation from which the standard deviation can thus be obtained. This method of Quick Estimate (Q. E.) was carried out for data ungrouped and grouped within the heating cycle in each of the seven tests by taking the first 10 failures of the original sample size of 20 and the first 5 of the constructed sample of size 10. The results in the form of the statistics are tabulated in Tables 10(a), 10(b), 11(a) and 11(b). The v^2 and t-test of the ungrouped first 10 and the first 5 quick estimates with the original ungrouped sample size of 20 can be seen in Table 6. As noted in this table, for the first 10 quick estimate procedure only Test B failed with Test G almost rejected. Both tests contain only 5 heating cycles maximum. The t-values appear to be inversely related to the average number of heating cycles. When only the first 5 failures out of sample of size 10 are used, all tests are rejected except C and D of 25 and 11 average heating cycles respectively. Another possible graphical procedure would be to sketch a straight line through the stepped fractile diagram or the points for the first 50% of the observations. However, this method would be subjective. For $n < 25$ the points lead to a line with systematic errors which would prevent utilizing the parameters estimated from the graph for further computations.

There is an analytical procedure[5] to follow when the distribution of data are censored by the observer or through experiment. The underlying assumption requires normal distribution of the observations. The calculations are long, involved and require the use of statistical tables[6]. The outlined procedure in the literature was modified to apply to the present condition where the data is known up to a point and unknown thereafter; this is a reverse procedure to that previously published. Application of this method was on the sample size 20 and size 10 with the data ungrouped and also grouped in the original heating cycle assuming the known point of truncation exists at 50% of the failures. The resulting statistics are shown in Tables 10(a), 10(b), 11(a) and 11(b). By comparing this calculated censored data with those obtained by the Quick Estimate methods, it is seen that there is good agreement between them. On the average, the more rigorous and analytical procedure provides the better estimate of the standard deviation and error; there were no zero values. This censored calculation offers the possibility of improving the estimates of the statistics when more observations are available (less censoring) while the quick estimate accuracy is not increased with additional failure points.

The latter method can also be improved with slight changes. For example, in the above analysis only the first 10 failures were employed although in many tests the 10th failure was part of a series of failures occurring at the same time. By including all the additional failures present with the 10th observation there is less censoring and a better estimate should be the result in censored distribution calculations.

REGRESSION ANALYSIS

The various factors of size, sample and number of heating cycles, form of the distribution and censored distributions have been discussed with no regard for the ultimate form of the data of life versus aging temperature. A plot of some of the statistics for all seven tests as calculated from the ungrouped data from samples of size 20 may be observed in Figure 5. There appears to be a distinct change in slope of the mean life curve at approximately 200°C. A possible reason for this reduction in rate of deterioration beyond 200°C is that the Formex insulation (aged) exhibits a softening point in that temperature region. Thus it follows that insulation subjected to vibration under a plastic condition would generally demonstrate a longer life. The possibility of errors in the independent variable, the test temperature, however, must not be disregarded. A variation of temperature among the coils will contribute to an error in the life of the coils. The short heating cycle of 4 hours for Tests C, D, E, F and G could introduce another error if the heating up period represents an appreciable percentage of the heating cycle with a relatively low deterioration rate. Corrosion of the copper is another factor.

The various calculated statistics are graphed on semi-log paper because, in the past, the life temperature curves for depicting a range of life[7], $\mu \pm 2\sigma$, for the various classes of insulation, A, B and H, have always been shown as straight lines on semi-logarithmic paper down to 1000 hours. When the end results appear in this form, it is advisable to analyze the original experimental data for a log-normal distribution. By definition, a variable has a logarithmic normal distribution if the logarithm of the variable is normally distributed. Logarithms of the failure time in hours were taken for each sample and the mean and the other statistics were calculated. A check as to whether the combined seven-test experimental data can be represented by a log-normal distribution was made graphically in a similar manner to that indicated in Figures 3(a) through 4(b). From a comparison of both sets of graphs it appears that either model for the failure distribution is satisfactory.

A further study into the possibility of whether or not the distribution of the failures is normal or log-normal was made by observing the relation between the variance and the standard deviation with the mean life. In Figure 6 the relation between the standard deviation and the mean life is seen to be a straight line with unit slope which indicates a linear relation. When this linearity prevails, it can be shown[8] that the distribution of failures in each sample can be log-normal. An analytical check of the probability of the log-normal distribution is shown in Table 2 for 0.8 class interval with and without a zero midpoint. The results of Table 2 allow for either the normal or log-normal distribution to obtain. This is not surprising since when the coefficient of variation, $s/\overline{x}$, is less than one-third, the middle failures can be represented by either form of distribution.

The question arises as to whether the mean life data may be best represented as a function of T°C, the log of T°K or 1/T°K. The latter two are plotted in Figure 7. Of the three curves of mean life outlined in Figures 5 and 7, the reciprocal absolute temperature plot for ungrouped sample size 20 was selected for further study and a regression analysis was made under the assumption of a log-normal distribution.

The independent variable, temperature, in each test was assumed a constant, thus grouping the data, enabling the variance of the dependent variable to be calculated. Before a regression analysis, Bartlett's test[9] is usually made to check the equality of variances associated with these points. If the differences between the variances are not significant, they all may be combined for an estimate of the variance for the entire seven-test data. Applying Bartlett's test to the seven points, it was found that the hypothesis of the equality of variance was not rejected. The data was analyzed on the basis of representation by one straight line. However, it was discovered that after the 95% confidence limit curves were drawn about the empirical line they did not encompass a sufficient number of the 95% confidence limits erected on the mean value points. It then became necessary to analyze the data on the basis of two intersecting straight lines. Application of Bartlett's test to tests A through D and tests D through G indicates that the respective groups of variances can be combined. Using these separate estimates for each of two lines, confidence limits on the mean are constructed for each point. The empirical solid lines were calculated and drawn as shown in Figure 8. The dashed lines represent the 95% confidence limits. As a check of this representation of data, it is to be noted that in all cases some portion of the 95% confidence limits constructed on the sample means lie within a region enclosed by the dashed lines. The dotted curves in this same figure represent the

tolerance limits for 95% of the population with a confidence level (probability) of 95%. The choice of these percentages are arbitrary for purposes of illustration. When the size of the samples is increased the confidence limits and tolerance limits (for ξ and the population) converge to the same value which for 95% is $\bar{x} \pm 1.96S$ (for ξ, $S = s/\sqrt{(n)}$; for population $S = s$).

SUMMARY

In any statistical analysis of experimental data it will be found frequently that there are many mathematical models which will adequately describe the data. The choice of a specific model should be governed by knowledge of the applicable physical laws and based on simplicity of application. A general statistical approach would also include design of experiments, sampling methods, number of observations, estimation of parameters, tests of significance and an investigation of agreement between the model and observations.

The failure of the coil samples with time did not make the choice of the mathematical model for the distribution definite on the basis of either a graphical or an analytical check. However, when the life-temperature characteristic is to be represented, the log-normal distribution is advisable.

The following statements are based on an analysis of the experimental data obtained from samples of size 20 (20 coils per test).

1. A sample of size 5 is adequate when there is no enforced cyclic grouping[+] providing there were at least 5 heating cycles.

2. A sample of size 20 is required when there is enforced cyclic grouping by the heating cycles provided there are more than 5 heating cycles.

3. When 50% censored testing is employed for a sample of size 20 more than 5 heating cycles are required. Applying 50% censored testing for a sample of size 10 requires at least 15 heating cycles. This is applicable to enforced cyclic groups.

[+] Enforced cyclic grouping exists when the failures are detected only at specific periods of time.

A more rigorous and definitive relation between the length of the heating cycle, maximum number of heating cycles and sample size to serve as a guide for future experimentation is outlined in Appendix B.

It would be economical to reduce the size of the samples and to reduce the number of observations by increasing the length of the heating cycle. This reduction plan cannot be carried out to the extreme where all failures occur during one or two cyclic intervals. A further economy in time can result if the failure data is censored after a certain percentage of failures have occurred. However, use of small sample censored data with few heating cycles to determine life temperature curves and the confidence limits on the mean and the tolerance limits on the future populations would not be as accurate or as narrow in scope as those in Figure 8.

The use of the censored distribution analysis provides similar data as the quick estimate procedure and, on the average, closer to the original ungrouped or cyclic data.

The lower tolerance limit can be used so as to be indicative of the maximum life for 2.5 percent of the future samples. It is felt that the tolerance limits on the samples are a better indication of the insulation to the ultimate user than the confidence limits on the mean of the samples.

It is conjectured that the breakup of the life temperature plot of Formex magnet wire into two distinct curves above 200°C may be due to a combination of increased plasticity, errors in test temperature and short length of heating cycle and corrosion of the copper wire. In view of this discontinuity in the life temperature curve, more experimental work is necessary to determine the feasibility of extrapolation of the high temperature results to normal operating temperatures.

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APPENDIX A

Nomenclature

c_x = coefficient of dispersion defined by $s_x/\bar{x}$.

i = index coefficient as in $\sum_{i=1}^{n} x_i$

l = number of multiples of the standard deviation taken to express tolerance limits. It is tabulated for various proportions of the population with assorted confidence coefficients.

n = number of observations in a sample; sample size.

n. d. f. = normal distribution function defined by

$$\frac{1}{(2\pi)^{\frac{1}{2}}\,\sigma}\; e^{-\frac{(t-\xi)}{2\sigma^2}} \qquad -\infty < t < \infty$$

s = estimate of the standard deviation, σ, defined by positive square root of the variance, s^2; together with the mean, ξ, determines the curve of normal distribution.

s_k = the estimate of standard deviation of the kth test.

s_x = the estimate of standard deviation of the mean, commonly called the standard error of the mean or just standard error, defined by $s/(n)^{1/2}$, the positive square root of the variance of the mean, s^2/n.

s^2 = the estimate of the variance of n observations about the mean, $\bar{x}$, defined by

$$\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} .$$

It can be calculated easily from $\frac{1}{n-1}\left(SS - \frac{S}{n}\right)$. The variance is used in further calculations and tests such as the variance ratio test.

t_p = a fractile of the t distribution and is tabulated; it is such that the probability of

$$\frac{\bar{x} - \xi}{s/(n)^{1/2}} < t_p \text{ is P.}$$

It converges to a normal distribution for very large samples, practically equal to u_p for samples larger than 30.

u = a standardized variable; a multiple of standard deviations.

u_k = a multiple of standard deviations of observations in a test from the mean of the test, defined by

$$\frac{x_{ik} - \bar{x}_k}{s_k}$$

u_p = a fractile, for a normal distribution and is tabulated; such that the probability of

$$\frac{x - \xi}{\sigma/(n)^{1/2}} < u_p \text{ is P.}$$

$v^2(f_1, f_2)$ = variance ratio $\frac{s_1^2}{s_2^2}$ where the degrees of freedom of numerator and denominator are f_1 and f_2 respectively and it is tabulated for various degrees of P, f_1 and f_2.

x_i = ith observation.

$\bar{x}$ = estimate of the mean defined by

$$\frac{1}{n} \sum_{i=1}^{n} x_i ;$$

Confidence limits on the mean are constructed in order that the following probability statement can be made. In a long series of samples each of n observations drawn from a normally distributed population, with parameters (ξ, σ^2), 95% of the intervals so determined by these confidence limits will include the fixed quantity ξ. The symbols ξ and σ^2 designate the theoretical mean and variance respectively.

$$\bar{x} - u_{P_2} \frac{\sigma}{(n)^{1/2}} < \bar{x} - u_{P_1} < \frac{\sigma}{(n)^{1/2}}$$

are the confidence limits, with confidence coefficients $P_2 - P_1$ (97.5% - 2.5%, chosen so as to make the interval bilateral and 95%). When the theoretical value of σ is unknown, the estimate, s, may be used and the limits for ξ are now

$$\bar{x} - t_{P_2} \frac{s}{(n)^{1/2}} < \bar{x} - t_{P_1} < \frac{s}{(n)^{1/2}}$$

employing the t distribution fractiles. Tolerance limits are used when it is desirable on the basis of one sample to predict within what limits a specified proportion of the future observations from the same population will fall. The two-sided limits are defined by

$$\bar{x} \pm l\, s$$

where $\bar{x}$ and s are calculated for a sample of size n. The values of l are tabulated for various confidence coefficients expressing the probability of the prediction, for various proportions of the distribution and for different sample sizes.

x_{ik} = ith observation of kth test.

C.D. = censored distribution.

N = total number of observations of several tests.

P = probability.

P(x) = probability that a variable $t \leq x$ for a cumulative distribution function

$$\frac{1}{(2\pi)^{1/2}\sigma} \int_{-\infty}^{x} e^{-\frac{(t-\xi)}{2\sigma^2}} \, dt.$$

20S, 10S, 5S, size of sample 20, 10 and 5 respectively.

T°C = temperature in degrees Centigrade.

T°K = temperature in degrees Kelvin.

Q.E. = quick estimate procedure.

ξ = projected mean of population which is estimated by sample mean $\bar{x}$.

$\varphi(u)$ = standard normal distribution function defined by

$$\frac{1}{2\pi} e^{-\frac{u^2}{2}}$$

of zero mean and unit variance.

σ = projected standard deviation of the population. For confidence limits see σ^2.

σ^2 = projected variance of the population, (σ_o^2 original data, σ_r^2 reduced sample size or cycle number data). Confidence limits for σ^2

$$\frac{s^2 f}{\chi^2_{P_2}} < \sigma^2 < \frac{s^2 f}{\chi^2_{P_1}}$$

employing the Chi square fractiles with confidence coefficients of P_2 and P_1 (97.5% - 2.5%) are chosen so as to make the interval bilateral and 95%. The confidence limits for σ are obtained by taking the square root of those for σ^2.

χ^2 = Chi square test symbol.

$\Phi(u)$ = standardized normal cumulative distribution function defined by

$$\frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2}} du \quad \text{with } u = \frac{x - \xi}{\sigma}.$$

APPENDIX B

Extension of Rounded Error Results

Closely allied to the problem of increase cycle length is the study[10] of the effect of rounding data. The applicable conclusions reached are as follows:

a. An unbiased estimate of ξ is given by either $\bar{x}$ or $\bar{x}_R$ where $\bar{x}_R$ is the rounded sample mean.

b. Whenever small sample size data is handled for variance ratio tests, t-tests and confidence limits a rough indication of the relation between the rounding interval and the sample size is as follows (the last two values obtained by extrapolation of (20) of reference 10):

for	$n \geqslant 2$	$a \leqslant 1/200$
	$n \geqslant 5$	$a \leqslant 0.5$
	$n \geqslant 10$	$a \leqslant 1.5$
	$n \geqslant 20$	$a \leqslant 2.4$

on the condition that the P is $\leqslant .001$ of $s_R^2 = 0$

where

$$a = \frac{\omega}{\sigma}$$

ω = the rounding interval

σ = population standard deviation may be obtained from estimate of

$$\sigma^2 = (s_R^2 - \frac{\omega^2}{12}).$$

s_R^2 = rounded sample variance.

c. A general recommendation that ω be taken less than $\frac{\sigma}{3}$, or, better less than $\frac{\sigma}{4}$ to limit the inevitable loss of precision in estimates of the parameters.

These results are now to be applied to heat aging, seven-test data, where the coil failures are grouped by the heating cycle characteristic with 85% failures on the average occurring at the start of a cycle.

For example, this grouping of the data is illustrated with part of test E abstracted from Table 1(a). The first six failures are at 16, 17, 18 with 3 at 20 hours. They may be grouped by cycles with one in the 4th cycle and 5 in the 5th cycle where the cycles are defined by

$$12 \text{ hours} < \text{4th cycle} \leq 16 \text{ hours}$$
$$16 \text{ hours} < \text{5th cycle} \leq 20 \text{ hours}$$

with the cycle length equal to 4 hours corresponding to a rounding interval ω.

From Figure 6:

$$s = 0.24\,\bar{x}$$

with

$$\omega = a\,\sigma \approx \quad as = a\,(0.24\bar{x})$$

ω	$\sigma/4$	$\sigma/3$	1.5σ	$2.4\,\sigma$
a	1/4	1/3	1.5	2.4
ω	$0.06\bar{x}$	$0.08\bar{x}$	$0.36\bar{x}$	$0.576\bar{x}$

From Table 1(b) and

$$\frac{1}{k}\sum_{i=1}^{k}\frac{x_{ni}-\bar{x}_i}{x_{ni}}$$

the average

$$\frac{x_n - \bar{x}}{x_n} = 0.3$$

where

$k = 1$ to 7 corresponding to tests A - G.

x_{ni} = nth failure of the ith test

$\bar{x}_i$ = mean of the ith test.

therefore,

$$x_n = \frac{\bar{x}}{0.7}$$

By definition, the maximum number of heating cycles

$$\text{H.C. max.} = \frac{x_n}{\omega}$$

(when this comes to a fractional part of a cycle it is raised to the next highest integer).

Substituting the values of ω listed above

a	1/4	1/3	1.5	2.4
H. C. max.	24	18	4	3

By applying the estimate of $\sigma^2 = s^2_R - \frac{\omega^2}{12}$ to the 20S data grouped into the original, two and four times the original heating cycle, as listed in Tables 6, 7(a) and (b) respectively, σ may be computed and then $a = \frac{\omega}{\sigma}$, (listed in Table 12). A plot of the calculated a of these tests under various cyclic groupings is shown in Figure 9 as a function of the H. C. max. In this figure it can be seen that some loss of precision in the estimates of the parameters of almost all of the tests can be expected. A line has been drawn through three of the values listed in the table above. The upper limit of a = 2.4, the dividing line for data roughly acceptable for application of approximate standard procedures, rejects 3 cycle data almost unanimously. Test F of 5 cycles length is almost rejected which would implicate five additional other tests of 5 or less heating cycles. This condition is in excellent agreement with conclusion 2 of this report. If, with 50% censored data, the effective sample size is taken to be 50%, the actual sample grouping requirements are 5 cycles and 12 cycles for size 20 and 10 samples respectively corresponding to a = 1.5, a = 0.5, in Figure 9. These values are in good agreement with conclusion 3 of this report.

ILLUSTRATIONS

FOR

NRL MEMORANDUM REPORT NO. 243

TABLE 1(a)

ORIGINAL EXPERIMENTAL DATA ON 20[1] COILS

Ungrouped Failures In Hours

Coil Tests

I.D.[2] No.	A	B	C	D	E	F	G
1	601	200	60	28	16	16	11
2	800	200	64	28	17	16	12
3	900	258	72	32	18	20	12
4	1000	300	73	32	20	20	12
5	1100	300	84	32	20	20	12
6	1100	300	84	40	20	20	12
7	1100	300	88	40	28	20.75	12
8	1100	300	88	40	32	21	12
9	1200	300	92	40	32	22.5	12
10	1200	399	104	40	32	24	16
11	1266	400	104	40	32	24	16
12	1274	400	108	40	33	24	16
13	1300	400	112	40	36	24	16
14	1300	400	116	42	36	24	16
15	1300	400	120	48	36	28	16
16	1300	400	124	52	36	28.5	16
17	1358	400	124	60	36	30	16
18	1400	500	132	60	40	32	19
19	1510	500	136	72	44	34	19
20			136	76	65[3]	59.5[3]	20

(1) Test samples A and B contained 19 coils.

(2) Identification number for chronological order.

(3) Observation rejected as "outlier".

TABLE 1(b)

GENERAL DATA ON ALL FORMEX COIL TESTS

Tests Completed on 20 Coils

Cyclic Heat Aging With Humidity

Test	Temp. °C	Life In Hours Min.	Max.	Mean	Heating Cycle Hours
A(1)	140	601	1510	1164	100
B(1)	160	200	500	350	100
C	180	60	136	101	4
D	200	28	76	44.1	4
E(2)	220	16	44	29.7	4
F(2)	250	16	34	23.6	4
G	270	11	20	14.7	4

(1) Size of sample was 19.

(2) Data on 19 because of rejection of "outlier".

TABLE 2

CHECKS ON DISTRIBUTIONS OF FAILURES OF THE COMBINED SEVEN TEST DATA USING THE CHI^2 TEST

		Class Interval Midpoints	Class Intervals	Degrees of Freedom	$\chi^2_{0.95}$	χ^2_{cal}
I.	**Ungrouped 7 - 20 Coil Tests**					
A.	Normal					
	1. Theoretical(9)	not zero	0.4	7	14.1	16.9
		not zero	0.8	3	7.82	3.98+
		zero	0.4	6	12.6	11.4+
		zero	0.8	2	5.99	4.96+
	2. Actual(10)	not zero(1)	0.8	3	7.82	3.56+
		zero(2)	0.8	2	5.99	4.91+
B.	Log-Normal					
	1. Theoretical(9)	not zero	0.4	6	12.6	21.6
		not zero	0.8	2	5.99	4.60+
		zero	0.4	7	14.1	11.3+
		zero	0.8	2	5.99	7.01
	2. Actual(10)	not zero(3)	0.8	2	5.99	3.91+
		zero(4)	0.8	2	5.99	8.12
II.	**Grouped Within Original Heating Cycle, 7 - 20 Coil Tests**					
A.	Normal(10)	not zero(5)	0.8	3	7.82	12.0
		zero(6)	0.8	2	5.99	7.75
B.	Log-Normal(10)	not zero(7)	0.8	3	7.82	7.20+
		zero(8)	0.8	2	5.99	3.51+

Note:

	$\bar{x}$	s^2
(1)	-0.03	0.98
(2)	0.06	0.96
(3)	-0.07	0.96
(4)	0.05	1.02
(5)	-0.04	1.09
(6)	0.00	1.14
(7)	-0.02	1.00
(8)	0.04	1.08

(9) Check of the distribution of failures against a theoretically normal distribution of zero mean and unit variance.

(10) Check of the distribution of failures against a normal distribution of the same mean and variance as the actual failures.

\+ Satisfactory check.

TABLE 3

CONSTRUCTED DATA FOR TESTS OF SAMPLE SIZE 10 AND SIZE 5

Ungrouped Failures in Hours

I.D.[1] No.	A[2]	B	C	D	E	F	G
1	601	200	72	28	16	16	11
2	1000	300	73	32	18	16	12
3	1100	300	88	32	20	20	12
4	1200	399	92	40	20	20	12
5	1266	400	104	40	20	20	12
6	1300	400	104	40	32	20.75	16
7	1300	400	120	42	36	21	16
8	1358	400	124	48	36	22.5	16
9	1400	500	136	52	40	24	19
10	1510	500	136	60	44	32	19
1	601	300	73	28	18	16	12
2	1000	300	104	40	20	20	12
3	1200	399	102	40	36	20.75	16
4	1266	400	136	52	36	24	16
5	1300	500	136	60	40	32	19

(1) Identification number for chronological order.

(2) Coil Tests A-G.

TABLE 4

REDUCTION IN SAMPLE SIZE - UNGROUPED DATA STATISTICS

Test	Mean Values			Variance			Standard Deviation		
	20S[1]	10S[2]	5S[3]	20S	10S	5S	20S	10S	5S
A	1164	1204	1073	47,200	65,610	83,790	217	256	289
B	350	380	380	7,410	8,440	6,895	86.0	91.9	82.1
C	101	105	114	575	564	668	23.9	23.8	25.8
D	44.1	41.4	44.0	181	96.2	152	13.5	9.81	12.3
E	29.7	28.2	30.0	73.0	109	104	8.54	10.4	10.2
F	23.6	21.2	22.6	24.8	20.4	34.8	4.99	4.51	5.90
G	14.7	14.5	15.0	7.93	9.39	9.00	2.82	3.06	3.00

Test	Coefficient of Variation			Standard Error			Average Number of Cycles		
	20S	10S	5S	20S	10S	5S	20S	10S	5S
A	0.187	0.213	0.266	49.7	80.9	129	11.2	12.0	10.7
B	0.246	0.242	0.216	19.8	29.0	36.7	3.5	3.8	3.8
C	0.237	0.227	0.217	5.35	7.52	11.5	25.2	26.2	28.5
D	0.306	0.237	0.279	3.02	3.10	5.50	11.0	10.4	11.0
E	0.288	0.369	0.340	2.02	3.29	4.55	7.42	7.05	7.50
F	0.211	0.221	0.261	1.18	1.43	2.64	5.90	5.30	5.65
G	0.192	0.211	0.200	0.63	0.97	1.34	3.68	3.63	3.75

(1) Sample of Size 20.

(2) Sample of Size 10.

(3) Sample of Size 5.

TABLE 5

SAMPLE SIZE CHECK

Variance Ratio Test (v^2) of Ungrouped 10S, 5S, 1st 10 Q. E.,[4] 1st 5 Q. E.
Compared With Ungrouped 20S

Test	10S	5S	1st 10 Q. E.	1st 5 Q. E.
A	1.39	1.78	(((1.97)))	1.54
B	1.14	1.07	1.35	1.35
C	1.02	1.16	1.67	1.57
D	1.88	1.19	(2.83)	(((2.83)))
E	1.49	1.43	(((1.97)))	(18.2)
F	1.22	1.41	1.55	1.55
G	1.18	1.14	(((2.05)))	(∞)
v^2 (f_1, f_2)	(19, 9)[3]	(19.4)[3]	(19, 19)[3]	(19, 9)[3]

Test For Equality of the Means of 10S, 5S, 1st 10, 1st 5 by the t-test
Compared with Ungrouped 20S

Test	10S	5S	1st 10 Q. E.	1st 5 Q. E.
A	-0.36	0.78	-0.43	(((-1.31)))
B	-0.83	-0.70	(((-1.70)))	(((-1.43)))
C	-0.42	-1.07	-0.34	-0.30
D	0.63	0.15	1.17	0.89
E	0.42	-0.07	0.14	(3.13)
F	1.33	0.63	0.61	((1.74))
G	0.18	-0.21	-1.19	(3.01)
t(f)	(28)[5]	(23)[5]	(28)[5]	(28)[5]

Note:

(1) Underlined variance ratios are those wherein the numerator contained the variance from the ungrouped 20 samples.

(2) The variance ratios and t-values which are in the single, double and triple parentheses exceed the 5%, 10% and 20% levels respectively.

(3) For tests A, B, E and F, v^2 (18, 9) v^2 (18, 4) and v^2 (18, 18) used.

(4) Quick Estimate procedure.

(5) For tests A, B, E and F, t (27) and t (22) used.

TABLE 6

DATA ON SAMPLES OF SIZE 20, 10 AND 5

Grouped Into Cycles of the Original Heating Period

TEST A

Cycle	Midpoint Hrs.	Failures 20S(1)	Failures 10S(2)	Failures 5S(3)
7	650	1	1	1
8	750	1		
9	850	1		
10	950	1	1	1
11	1050	4	1	
12	1150	2	2	1
13	1250	6	3	2
14	1350	2	1	
16	1550	1	1	

TEST B

Cycle	Midpoint Hrs.	Failures 20S	Failures 10S	Failures 5S
2	150	2	1	
3	250	7	2	2
4	350	8	5	2
5	450	2	2	1

TEST C

Cycle	Midpoint Hrs.	Failures 20S	Failures 10S	Failures 5S
15	58	1		
16	62	1		
18	70	1	2	1
19	74	1		
21	82	2		
22	86	2	1	
23	90	1	1	
26	102	2	2	1
27	106	1		
28	110	1		
29	114	1		
30	118	1	1	1
31	122	2	1	
33	130	1		
34	134	2	2	2

Cycle	Midpoint Hrs.	TEST D Failures 20S	TEST D Failures 10S	TEST D Failures 5S	TEST E Failures 20S	TEST E Failures 10S	TEST E Failures 5S	TEST F Failures 20S	TEST F Failures 10S	TEST F Failures 5S	TEST G Failures 20S	TEST G Failures 10S	TEST G Failures 5S
3	10										9	5	2
4	14				1	1	1	2	2	1	8	3	2
5	18				5	4	1	4	3	2	3	2	1
6	22							8	4	1			
7	26	2	1	1	1			1					
8	30	3	2		4	1		2	1	1			
9	34				6	2	2	2					
10	38	8	3	2	1	1	1						
11	42	1	1		1	1							
12	46	1	1										
13	50	1	1	1									
15	58	2	1	1									
16	62												
18	70	1											
19	74	1											

Note:
(1) Sample size 20.
(2) Sample size 10.
(3) Sample size 5.

TABLE 7 (a)

DATA ON SAMPLES OF SIZE 20 AND 10

Grouped Into Cycles Twice as Large as Original Heating Period

Test A Cycle	Test A Midpoint Hours	Test A Failures 20S[(1)]	Test A Failures 10S[(1)]	Test B Cycle	Test B Midpoint Hours	Test B Failures 20S	Test B Failures 10S
4	700	2	1	1	100	2	1
5	900	2	1	2	300	15	7
6	1100	6	3	3	500	2	2
7	1300	8	4				
8	1500	1	1				

Cycle	Test C Midpoint Hours	Test C Failures 20S	Test C Failures 10S	Test D Failures 20S	Test D Failures 10S	Test E Failures 20S	Test E Failures 10S	Test F Failures 20S	Test F Failures 10S	Test G Failures 20S	Test G Failures 10S
2	12					1	1	2	2	17	8
3	20					5	4	12	7	3	2
4	28			5	3	5	1	4	1		
5	36			8	3	7	3	1			
6	44			2	2	1	1				
7	52			1	1						
8	60	2		2	1						
9	68	1	1	1							
10	76	1	1	1							
11	84	4	1								
12	92	1	1								
13	100	2	2								
14	108	2									
15	116	2	1								
16	124	2	1								
17	132	3	2								

(1) Sample Size 20.

(2) Sample Size 10.

TABLE 7(b)

DATA ON SAMPLES OF SIZE 20 AND 10

Grouped Into Heating Periods Four Times As Large As Original Heating Cycle

Test A				Test B			
Cycle	Midpoint Hours	Failures 20S(1)	Failures 10S(2)	Cycle	Midpoint Hours	Failures 20S	Failures 10S
2	600	2	1	1	200	17	8
3	1000	8	4	2	600	2	2
4	1400	9	5				

	Test C			Test D		Test E		Test F		Test G	
Cycle	Midpoint Hours	Failures 20S	Failures 10S	Failures 20S	Failures 10S	Failures 20S	Failures 10S	Failures 20S	Failures 10S	Failures 20S	Failures 10S
1	8					1	2	2	2	17	8
2	24			5	3	10	4	16	8	3	2
3	40			10	5	8	4	1			
4	56	2		3	2						
5	72	2	2	2							
6	88	5	2								
7	104	4	2								
8	120	4	2								
9	136	3	2								

(1) Sample Size 20.

(2) Sample Size 10.

TABLE 8(a)

Statistics on Increase in Heating Cycle Length for Sample of Size 20

	Mean Values			Variance			Standard Deviation		
Test	Orig.[1]	2X[2]	4X[3]	Orig.	2X	4X	Orig.	2X	4X
A	1124	1132	1188	53,300	57,600	75,200	231	240	274
B	302	300	242	7,000	8,800	15,900	83.5	94.4	126
C	99	99	100	563	563	605	23.7	23.6	24.6
D	44.2	40.8	41.6	117	224	213	10.8	15.0	14.6
E	28.1	28.8	29.9	69.2	66.8	91.2	8.32	8.16	9.55
F	22.6	21.7	23.1	33.7	16.7	42.8	5.81	4.09	6.55
G	12.8	12.8	10.4	8.60	8.84	34.3	2.92	2.96	5.85

	Coefficient of Variation			Standard Error			Average Number of Cycles		
Test	Orig.	2X	4X	Orig.	2X	4X	Orig.	2X	4X
A	0.203	0.212	0.231	53.0	55.0	62.8	11.2	5.66	2.97
B	0.276	0.314	0.520	19.2	2.17	29.0	3.02	1.50	0.605
C	0.242	0.238	0.246	5.31	5.23	5.51	24.8	12.4	6.25
D	0.244	0.368	0.351	2.42	3.36	3.34	11.1	5.10	2.60
E	0.293	0.283	0.320	1.97	1.87	2.25	7.02	3.60	1.87
F	0.257	0.189	0.283	1.37	0.97	1.54	5.66	2.71	1.45
G	0.228	0.231	0.562	0.65	0.60	1.31	3.20	1.60	0.65

(1) Original heating cycle length.

(2) Twice original heating cycle length.

(3) Four times original heating cycle length.

TABLE 8(b)

STATISTICS ON INCREASE IN HEATING CYCLE LENGTH FOR SAMPLE OF SIZE 10

Tes	Mean Values			Variance			Standard Deviation		
	Orig.(1)	2X(2)	4X(3)	Orig.	2X	4X	Orig.	2X	4X
A	1140	1140	1160	62,200	77,500	78,400	249	278	280
B	330	320	280	8,450	12,900	28,500	91.9	114	169
C	105	102	104	710	580	578	27.0	24.1	23.8
D	39.6	39.2	38.4	105	116	343	10.2	10.8	17.8
E	26.0	26.4	37.2	114	125	159	10.6	11.2	12.6
F	19.2	19.1	20.8	21.4	20.6	45.5	4.64	4.54	6.75
G	12.8	13.6	11.2	10.8	11.4	45.5	3.29	3.38	6.75

Test	Coefficient of Variation			Standard Error			Average Number of Cycles		
	Orig.	2X	4X	Orig.	2X	4X	Orig.	2X	4X
A	0.219	0.244	0.241	78.7	87.8	88.5	11.4	5.20	2.65
B	0.278	0.356	0.603	29.0	36.0	53.4	3.3	1.60	0.70
C	0.256	0.238	0.228	8.53	7.66	7.52	26.3	12.7	6.50
D	0.259	0.276	0.463	3.22	3.41	5.62	9.90	4.90	2.46
E	0.408	0.424	0.463	3.34	3.54	3.98	6.50	3.30	1.70
F	0.241	0.238	0.327	1.47	1.43	2.13	4.80	2.34	1.30
G	0.238	0.248	0.603	1.04	1.07	2.13	3.20	1.70	0.70

(1) Original heating cycle length.

(2) Twice original heating cycle length.

(3) Four times original heating cycle length.

TABLE 9

CHECK ON NUMBER OF CYCLES

A. Sample of Size 20

Test	Variance Ratio Test (v^2)			Equality of Means (t-test)		
	Cycles			Cycles		
	Original	1/2	1/4	Original	1/2	1/4
A	1.08	1.08	1.30	0.43	-0.11	-0.78
B	$\underline{1.06}$	1.26	((2.27))	((1.71))	0.07	((1.72))
C	1.02	1.00	1.07	0.27	0.00	0.13
D	$\underline{1.63}$	(((1.91)))	(2.08)	-0.03	0.82	0.80
E	$\underline{1.05}$	1.04	1.32	0.59	-0.26	-0.62
F	1.36	((($\underline{2.02}$)))	1.27	0.55	0.55	-0.25
G	1.09	1.03	(3.99)	(2.10)	0.00	(((1.62)))
$v^2(f_1, f_2)$	(19, 19)[3]	(19, 19)[3]	(19, 19)[3] t(f)	(38)[4]	(38)[4]	(38)[4]

B. Sample of Size 10

Test	Variance Ratio Test (v^2)			Equality of Means (t- test)		
	Cycles			Cycles		
	Original	1/2	1/4	Original	1/2	1/4
A	$\underline{1.06}$	1.25	1.26	0.54	0.00	-0.17
B	1.01	1.52	((3.38))	1.14	0.21	0.82
C	1.26	$\underline{1.22}$	$\underline{1.23}$	0.00	0.17	0.09
D	1.09	1.10	((3.27))	0.37	0.09-	0.18
E	1.05	1.09	1.40	0.44	-0.09	(-2.14)
F	1.05	$\underline{1.04}$	2.12	0.92	0.05	-0.62
G	1.15	1.05	(4.22)	1.13	-0.54	0.67
$v^2(f_1, f_2)$	(9, 9)	(9, 9)	(9, 9) t(f)	(18)	(18)	(18)

(1) Underlined variance ratios are those where the numerator contains the variance from the original size 20 or 10.

(2) The variance and t-values ratios which are in single, double and triple parentheses exceeds the 5%, 10%, and 20% significance levels respectively.

(3) For tests A, B, E, F, v^2, (18, 18) are used.

(4) For tests A, B, E, F, t (36) used.

TABLE 10(a)

STATISTICS ON QUICK ESTIMATE AND CENSORED DISTRIBUTIONS

Procedure on Sample of Size 20 - First 10 Ungrouped Failures

Test	Mean Values 20S (1)	Q.E. (2)	C. D. (3)	Variance 20S	Q.E.	C. D.	Standard Deviation 20S	Q.E.	C. D.
A	1164	1200	1215	47,200	90,000	72,360	217	300	269
B	350	400	389	7,410	10,000	14,640	86.0	100	121
C	101	104	103	575	961	660	23.9	31	25.7
D	44.1	40	40.5	181	64	50.4	13.5	8	7.1
E	29.7	32	32	73	144	112	8.54	12	10.6
F	23.6	24	24.8	24.8	16	21.3	4.99	4	4.62
G	14.7	16	15.5	7.93	16	13.5	2.82	4	3.68

Test	Coefficient of Variation 20S	Q.E.	C.D.	Standard Error 20S	Q.E.	C.D.	Average Number of Cycles 20S	Q.E.	C.D.
A	0.187	0.250	0.222	49.7	67.2	76.2	11.2	12.0	12.2
B	0.246	0.250	0.311	19.8	23.0	32.4	3.50	4.00	3.89
C	0.237	0.298	0.250	5.35	6.94	6.90	25.2	26.0	25.7
D	0.306	0.200	0.175	3.02	1.79	2.03	11.0	10.0	10.2
E	0.288	0.375	0.331	2.02	2.69	2.92	7.42	8.00	8.00
F	0.211	0.167	0.186	1.18	0.90	1.25	5.90	6.00	6.20
G	0.192	0.250	0.238	0.63	0.90	0.96	3.68	4.00	3.88

(1) Sample size 20.

(2) Quick Esimate procedure.

(3) Censored distribution calculation.

TABLE 10(b)

STATISTICS ON QUICK ESTIMATE AND CENSORED DISTRIBUTION

Procedure on First 10 Cyclic Grouped Failures of Sample Size 20

Test	Mean Values			Variance			Standard Deviation		
	20S[1]	Q.E.[2]	C.D.[3]	20S	Q.E.	C.D.	20S	Q.E.	C.D.
A	1124	1150	1153	53,300	40,000	77,280	231	200	278
B	302	350	395	7,000	0	8,100	83.5	0	90
C	99	102	103	563	784	900	23.7	28	30
D	44.2	38	40.0	117	64	70.6	10.8	8	8.4
E	28.1	30	31.4	69.2	144	108	8.32	12	11.4
F	22.6	22	24.0	33.7	16	34.3	5.81	4	5.86
G	12.8	14	15.3	8.6	16	28.2	2.92	4	5.31

Test	Coefficient of Variation			Standard Error			Average Number of Cycles		
	20S	Q.E.	C.D.	20S	Q.E.	C.D.	20S	Q.E.	C.D.
A	0.203	0.174	0.240	53.0	44.7	75.8	11.2	11.5	11.5
B	0.276	0.00	0.228	19.2	0.00	24.3	3.02	3.50	4.00
C	0.242	0.274	0.292	5.31	6.25	8.20	24.8	25.5	25.7
D	0.244	0.210	0.210	2.42	0.79	2.31	11.1	9.50	10.0
E	0.293	0.400	0.383	1.97	2.68	3.08	7.02	7.50	7.85
F	0.257	0.182	0.244	1.37	0.90	1.57	5.66	5.50	6.00
G	0.228	0.311	0.347	0.65	0.90	1.41	3.20	3.50	3.82

(1) Sample size 20.

(2) Quick Estimate procedure.

(3) Censored distribution calculation.

TABLE 11(a)

STATISTICS ON QUICK ESTIMATE AND CENSORED DISTRIBUTION

Procedure on First Five Ungrouped Failures of Sample Size 10

Test	Mean Values 10S[1]	Mean Values Q.E.[2]	Mean Values C.D.[3]	Variance 10S	Variance Q.E.	Variance C.D.	Standard Deviation 10S	Standard Deviation Q.E.	Standard Deviation C.D.
A	1204	1270	1237	65,610	72,900	101,100	256	270	318
B	380	400	405	8,440	10,000	12,540	91.9	100	112
C	105	104	103	564	900	458	23.8	30	21.4
D	41.4	40	40.2	96.2	64	55.8	9.81	8	7.47
E	28.2	20	20.3	109	4	4.33	10.4	2	2.08
F	21.2	20	20.3	20.4	16	5.86	4.51	4	2.42
G	14.5	12	+	0.39	0	+	3.06	0	+

Test	Coefficient of Variation 10S	Coefficient of Variation Q.E.	Coefficient of Variation C.D.	Standard Error 10S	Standard Error Q.E.	Standard Error C.D.	Average Number of Cycles 10S	Average Number of Cycles Q.E.	Average Number of Cycles C.D.
A	0.213	0.213	0.257	80.9	85.3	129	12.0	12.7	12.4
B	0.242	0.250	0.277	29.0	31.6	44.6	3.8	4.0	4.05
C	0.227	0.288	0.208	7.52	9.48	8.31	26.2	26.0	25.7
D	0.237	0.200	0.186	3.10	2.53	2.95	10.4	10.6	10.1
E	0.369	0.100	0.103	3.29	1.63	0.87	7.05	5.0	5.06
F	0.221	0.200	0.120	1.43	1.26	1.00	5.30	5.0	5.06
G	0.211	0.00	+	0.97	0.0	+	3.63	3.0	+

\+ Could not be estimated as the value of an intermediate factor was not in the tables.

(1) Sample size 10.

(2) Quick Estimate procedure.

(3) Censored distribution calculation.

TABLE 11(b)

STATISTICS ON QUICK ESTIMATE AND CENSORED DISTRIBUTION

Procedure on First 5 Cyclic Grouped Failures on Sample Size 10

	Mean Value			Variance			Standard Deviation		
Test	10S (1)	Q.E. (2)	C.D. (3)	10S	Q.E.	C.D.	10S	Q.E.	C.D.
A	1140	1250	1308	12,200	40,000	108,900	249	200	330
B	330	350	389	8,450	0	13,460	91.9	0	116
C	105	102	103	710	784	506	27.0	28	22.5
D	39.6	38	39.7	105	64	79.2	10.2	8	8.9
E	26.0	18	19.2	114	0	10.9	10.6	0	3.20
F	19.2	18	19.8	21.4	16	16.3	4.64	4	4.04
G	12.8	10	11.7	10.8	0	3.35	3.29	0	1.83

	Coefficient of Variation			Standard Error			Average Number of Cycles		
Test	10S	Q.E.	C.D.	10S	Q.E.	C.D.	10S	Q.E.	C.D.
A	0.219	0.160	0.252	78.7	63.3	130	11.4	12.5	13.1
B	0.218	0.0	0.298	29.0	0	43.5	3.3	3.5	3.9
C	0.256	0.275	0.219	8.53	8.85	8.60	26.3	25.5	25.7
D	0.259	0.210	0.224	3.22	2.53	3.43	8.9	9.5	9.90
E	0.408	0.0	0.167	3.34	0	1.26	6.50	4.5	4.80
F	0.241	0.222	0.204	0.47	0.27	0.53	4.80	4.5	4.95
G	0.258	0.0	0.56	1.04	0	0.67	3.20	2.5	2.92

(1) Sample size 10.

(2) Quick Estimate procedure.

(3) Censored distribution calculation.

TABLE 12

CYCLIC GROUPING ANALOGY TO ROUNDING ERROR

I. Original Cycle Grouping

	A	B	C	D	E	F	G
σ^2	52500	6180	562	116	67.9	32.4	7.27
σ	229	78.5	23.8	10.8	8.24	5.70	2.70
$a = \omega/\sigma$	0.44	1.27	0.17	0.37	0.49	0.70	1.48
H. C. max.	16	5	34	19	11	9	5

II. Two Times Original Cycle Length

	A	B	C	D	E	F	G
σ^2	54300	5470	558	219	61.5	11.4	3.51
σ	234	73.9	23.6	14.8	7.85	3.38	1.88
$a = \omega/\sigma$	0.86	2.71	0.34	0.54	1.02	2.36	4.26
H. C. max.	8	3	17	10	6	5	3

III. Three Times Original Cycle Length

	A	B	C	D	E	F	G
σ^2	66900	2570	584	192	69.9	21.5	13.0
σ	248	50.9	24.2	13.9	8.35	4.64	3.61
$a = \omega/\sigma$	1.62	7.86	0.66	1.16	1.92	3.45	4.45
H. C. max.	4	2	9	5	3	3	2

	A and B Tests			C, D, E, F, G Tests		
	Orig.(1)	2X(2)	4X(3)	Orig.	2X	4X
ω	100	200	400	4	8	16
ω^2	10,000	40,000	160,000	16	64	256
$\omega^2/12$	825	3,330	13,300	1.33	5.33	21.3

with

$$\sigma^2 = s^2_R - \frac{\omega^2}{12}$$

(1) Original cycle grouping.
(2) Grouping in cycles two times original.
(3) Grouping in cycles four times original.

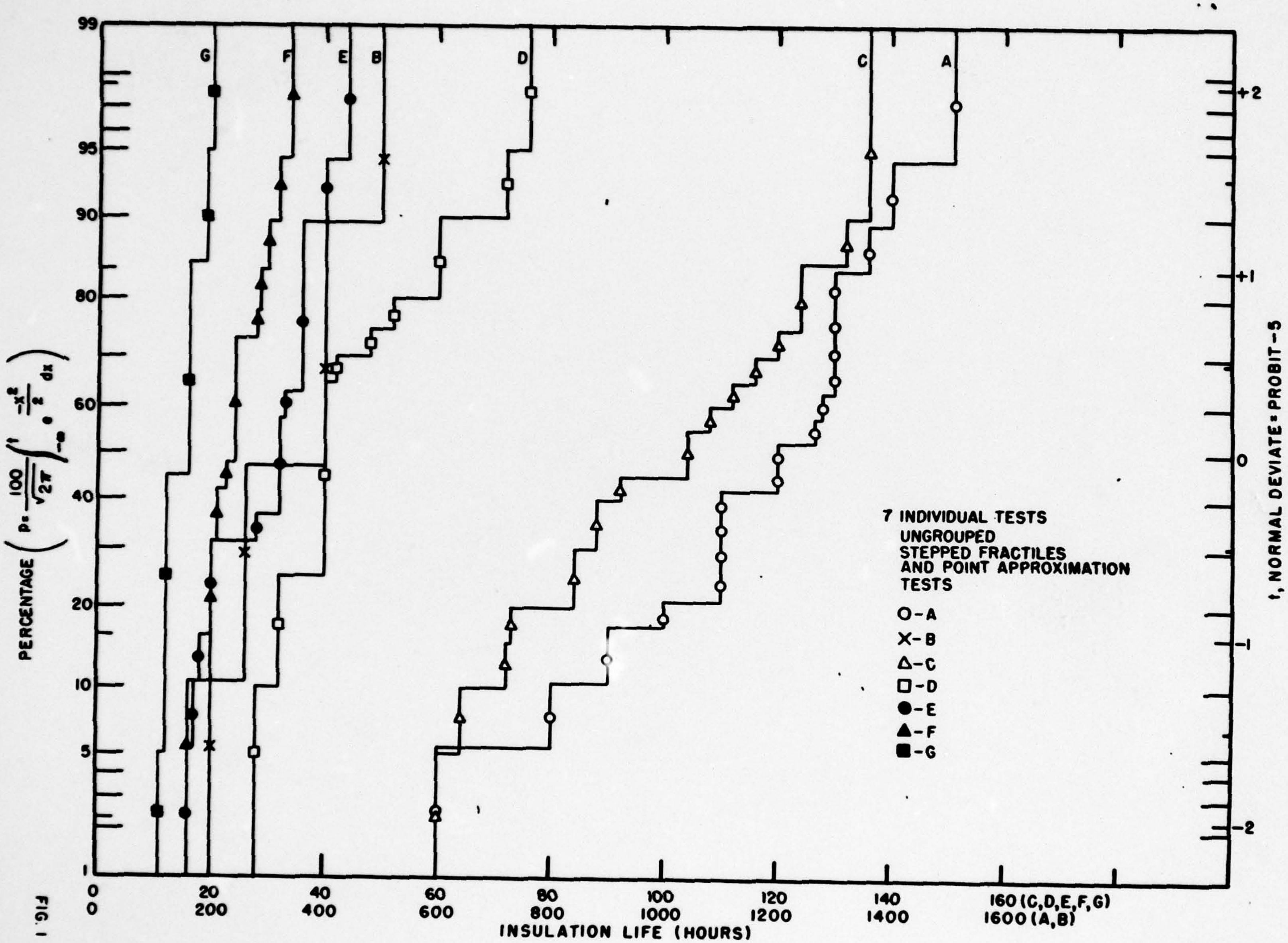

7 INDIVIDUAL TESTS
UNGROUPED
STEPPED FRACTILES
AND POINT APPROXIMATION
TESTS
O-A
X-B
△-C
□-D
●-E
▲-F
■-G
A
B
C
D
E
F
G
PERCENTAGE $\left(P = \frac{100}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{\frac{-x^2}{2}} dx\right)$
t, NORMAL DEVIATE = PROBIT - 5
INSULATION LIFE (HOURS)
160 (C,D,E,F,G)
1600 (A,B)

FIG. 1

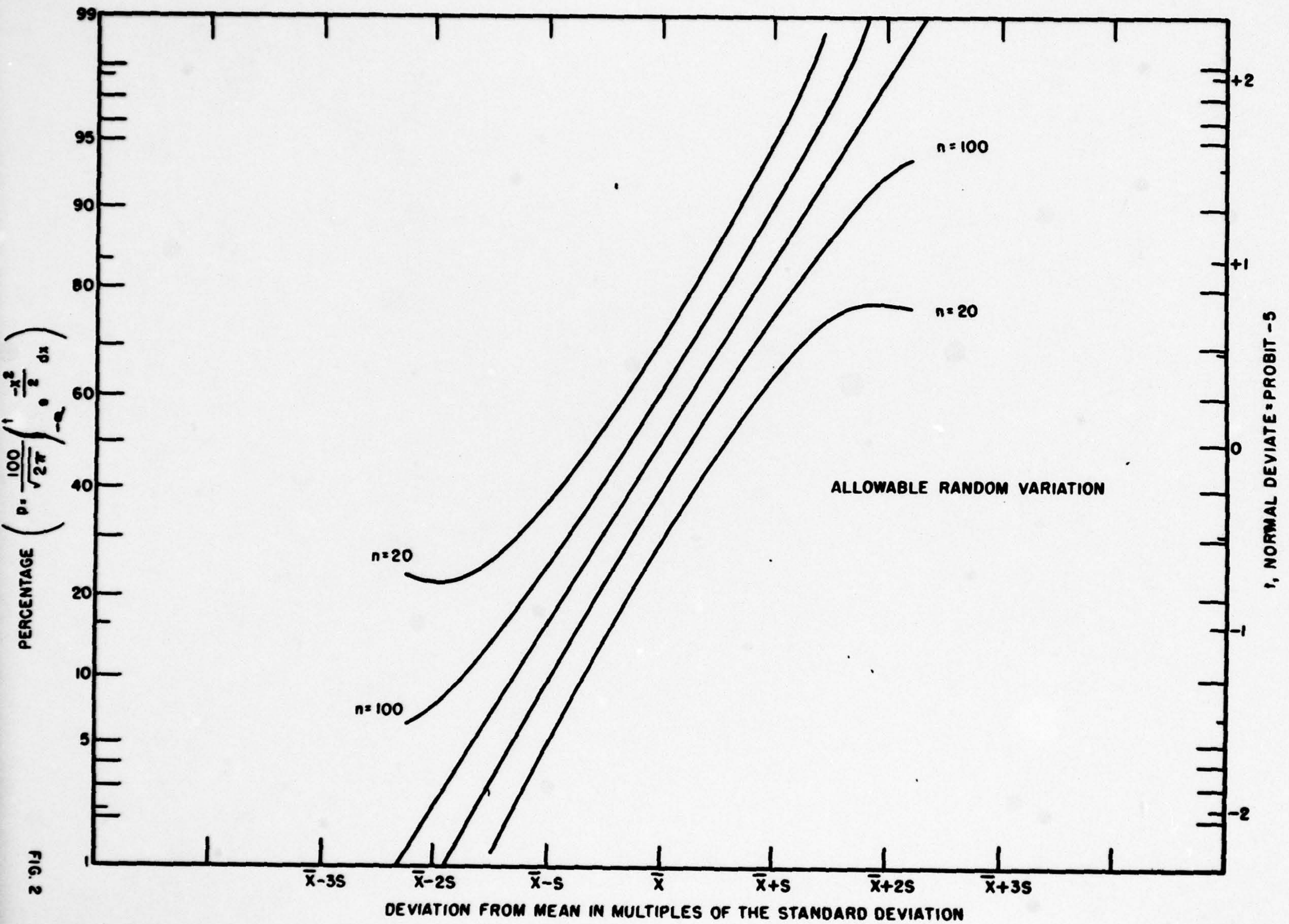
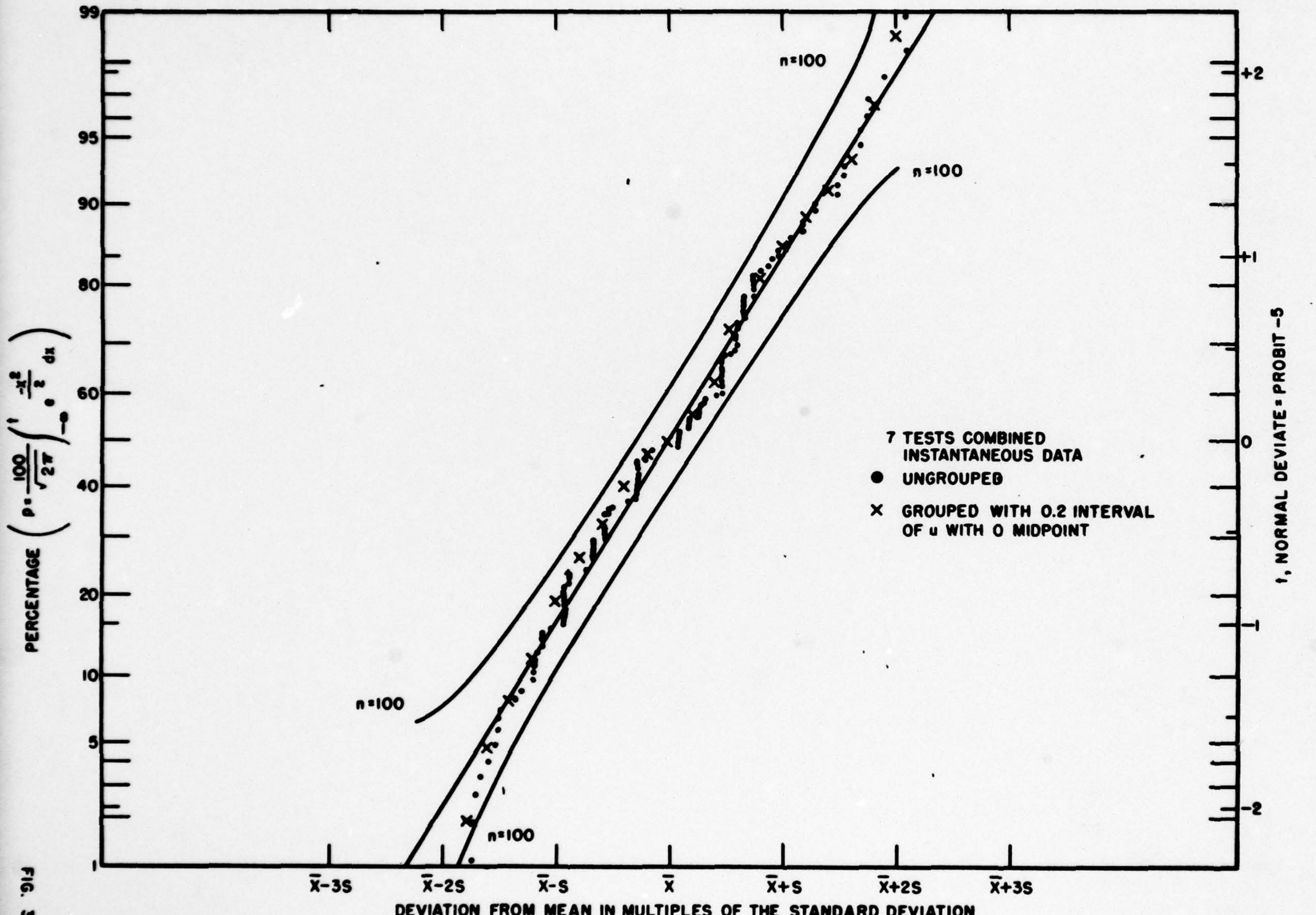

ALLOWABLE RANDOM VARIATION
n=20
n=100
n=20
n=100
PERCENTAGE $\left(P = \frac{100}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{\frac{-x^2}{2}} dx \right)$
99
95
90
80
60
40
20
10
5
1
$\bar{X}$-3S
$\bar{X}$-2S
$\bar{X}$-S
$\bar{X}$
$\bar{X}$+S
$\bar{X}$+2S
$\bar{X}$+3S
DEVIATION FROM MEAN IN MULTIPLES OF THE STANDARD DEVIATION
t, NORMAL DEVIATE = PROBIT -5
+2
+1
0
-1
-2

FIG. 2



PERCENTAGE $\left(p = \frac{100}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{\frac{-x^2}{2}} dx \right)$
99
95
90
80
60
40
20
10
5
1
t, NORMAL DEVIATE = PROBIT -5
+2
+1
0
-1
-2
n=100
n=100
n=100
n=100
7 TESTS COMBINED
INSTANTANEOUS DATA
● UNGROUPED
× GROUPED WITH 0.2 INTERVAL
OF u WITH 0 MIDPOINT
$\bar{X}$-3S
$\bar{X}$-2S
$\bar{X}$-S
$\bar{X}$
$\bar{X}$+S
$\bar{X}$+2S
$\bar{X}$+3S
DEVIATION FROM MEAN IN MULTIPLES OF THE STANDARD DEVIATION

FIG. 3A

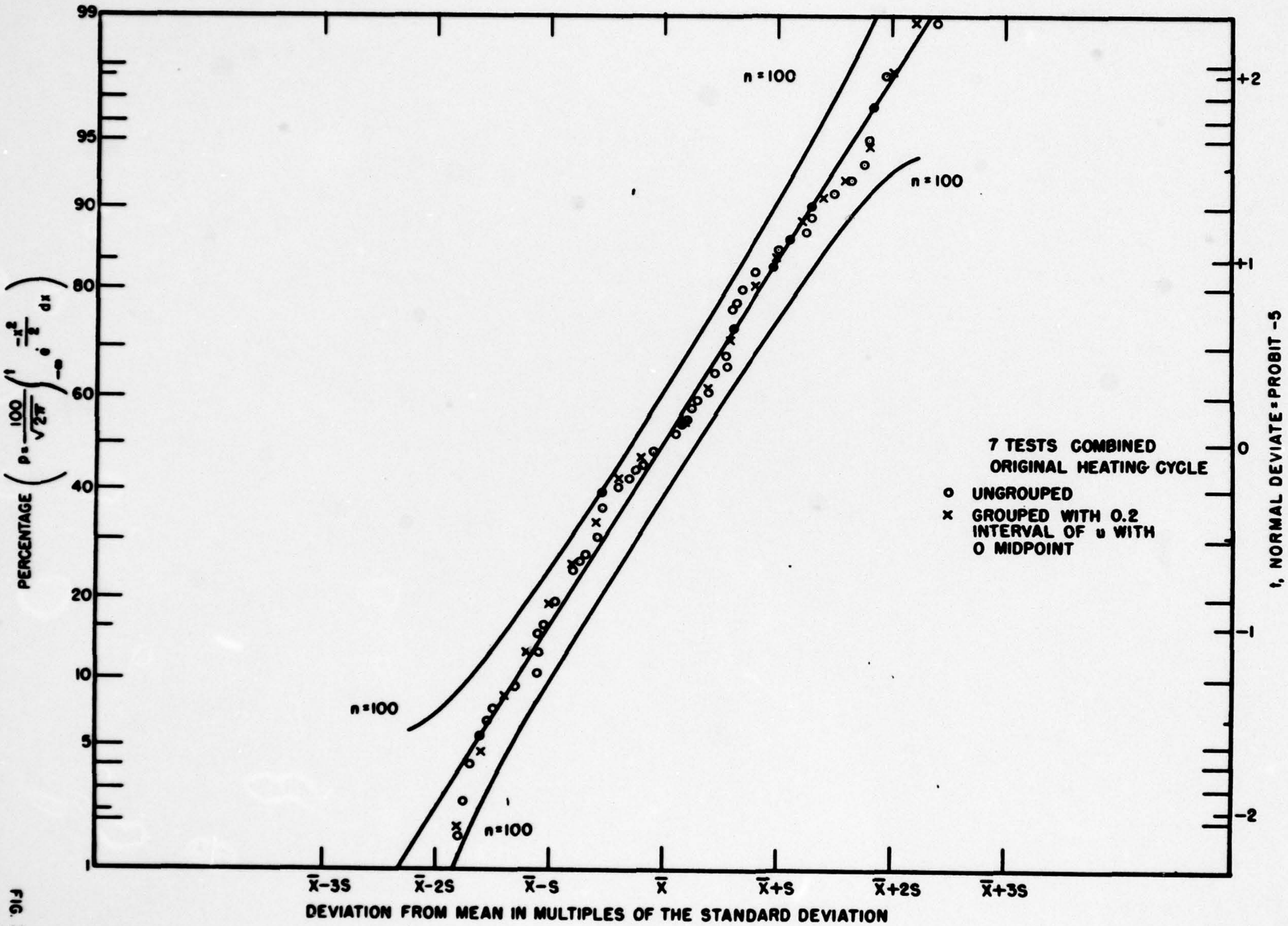
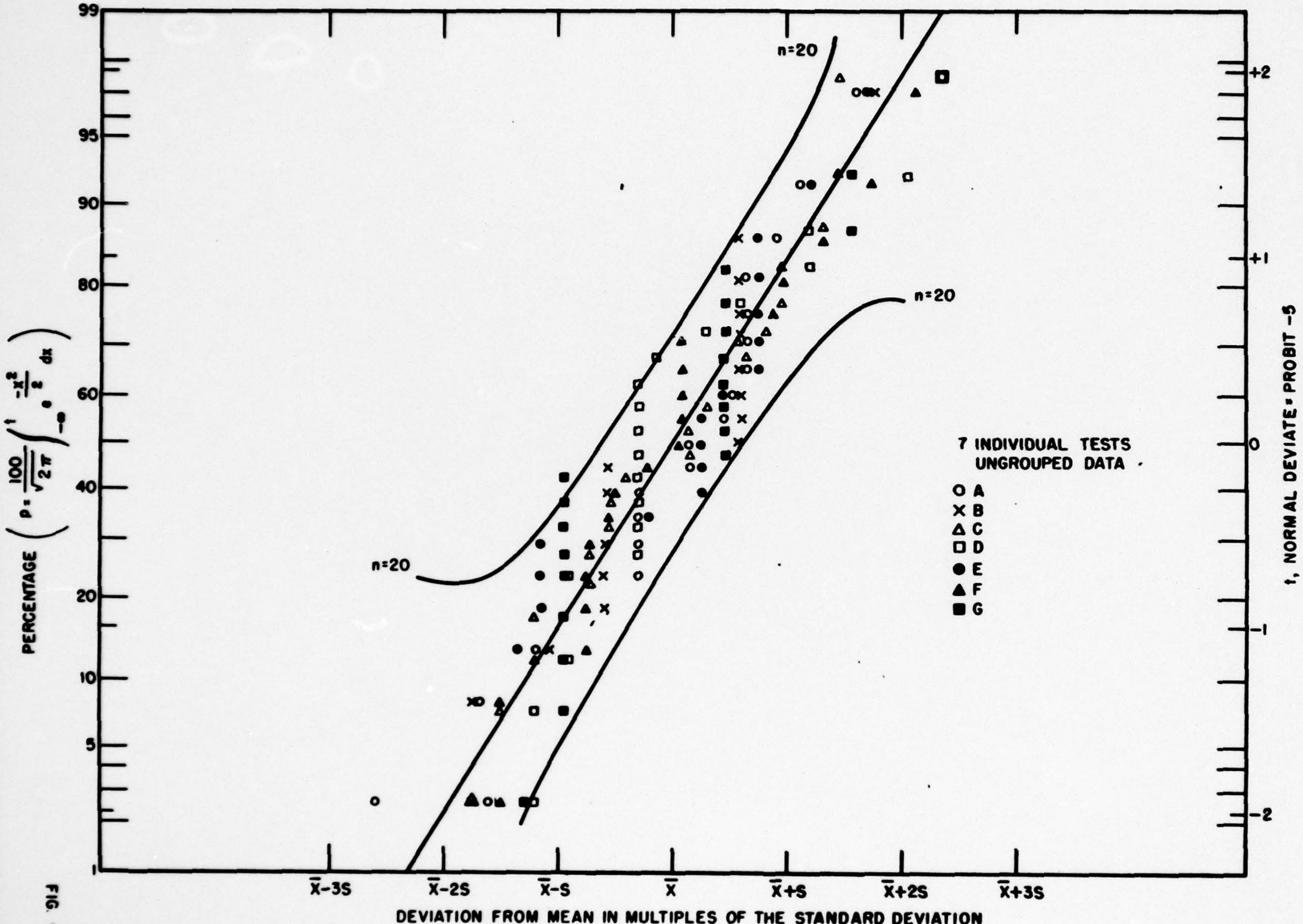
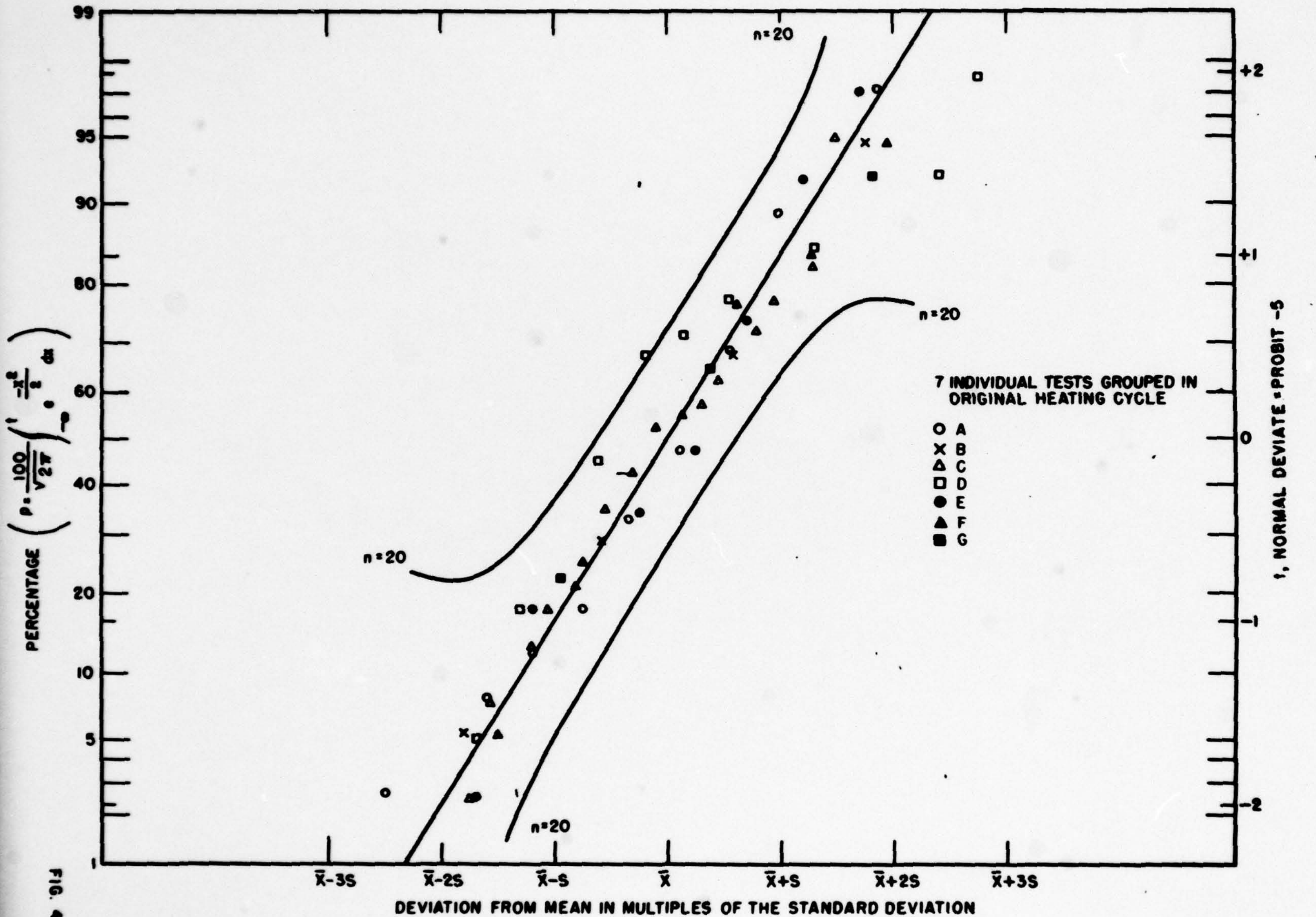

7 TESTS COMBINED
ORIGINAL HEATING CYCLE
o UNGROUPED
x GROUPED WITH 0.2 INTERVAL OF u WITH 0 MIDPOINT
n=100
n=100
n=100
n=100
PERCENTAGE $\left(p=\frac{100}{\sqrt{2\pi}}\int_{-\infty}^{t} e^{-\frac{x^2}{2}}dx\right)$
99
95
90
80
60
40
20
10
5
1
t, NORMAL DEVIATE=PROBIT -5
+2
+1
0
-1
-2
$\bar{X}$-3S
$\bar{X}$-2S
$\bar{X}$-S
$\bar{X}$
$\bar{X}$+S
$\bar{X}$+2S
$\bar{X}$+3S
DEVIATION FROM MEAN IN MULTIPLES OF THE STANDARD DEVIATION

FIG. 3B



PERCENTAGE $\left(P = \frac{100}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{\frac{-x^2}{2}} dx \right)$
99
95
90
80
60
40
20
10
5
1
n=20
n=20
n=20
7 INDIVIDUAL TESTS
UNGROUPED DATA
A
B
C
D
E
F
G
+2
+1
0
-1
-2
t, NORMAL DEVIATE = PROBIT -5
$\bar{X}-3S$
$\bar{X}-2S$
$\bar{X}-S$
$\bar{X}$
$\bar{X}+S$
$\bar{X}+2S$
$\bar{X}+3S$
DEVIATION FROM MEAN IN MULTIPLES OF THE STANDARD DEVIATION

FIG. 4A



PERCENTAGE $\left(P = \frac{100}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{\frac{-x^2}{2}} dx\right)$
t, NORMAL DEVIATE = PROBIT -5
DEVIATION FROM MEAN IN MULTIPLES OF THE STANDARD DEVIATION
7 INDIVIDUAL TESTS GROUPED IN ORIGINAL HEATING CYCLE
○ A
× B
△ C
□ D
● E
▲ F
■ G
n=20
n=20
n=20
n=20
99
95
90
80
60
40
20
10
5
1
+2
+1
0
-1
-2
$\bar{X}$-3S
$\bar{X}$-2S
$\bar{X}$-S
$\bar{X}$
$\bar{X}$+S
$\bar{X}$+2S
$\bar{X}$+3S

FIG. 4B

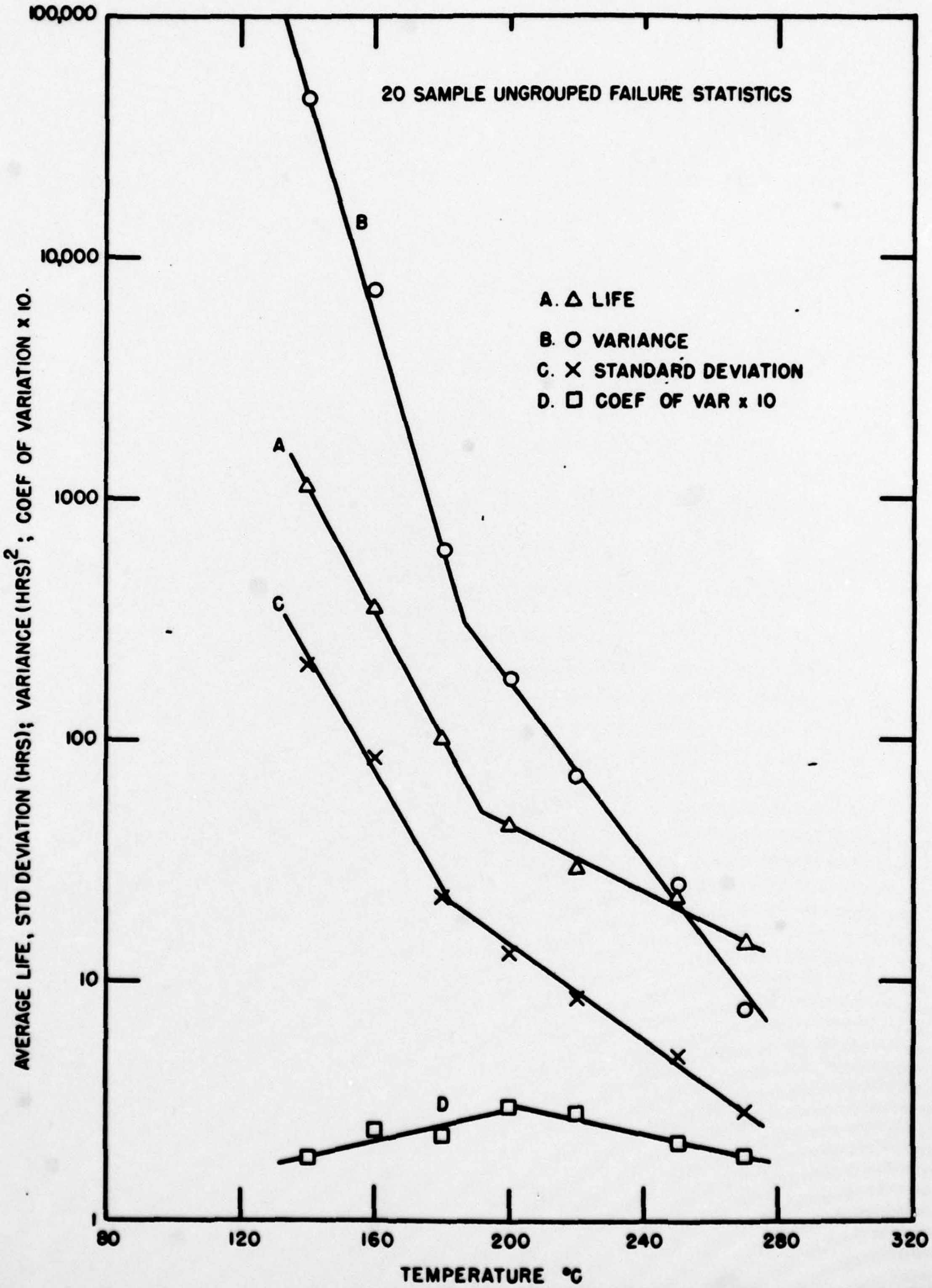

20 SAMPLE UNGROUPED FAILURE STATISTICS
A. △ LIFE
B. ○ VARIANCE
C. × STANDARD DEVIATION
D. □ COEF OF VAR x 10
A
B
C
D
AVERAGE LIFE, STD DEVIATION (HRS); VARIANCE (HRS)^2; COEF OF VARIATION x 10.
100,000
10,000
1000
100
10
1
80
120
160
200
240
280
320
TEMPERATURE °C

FIG. 5

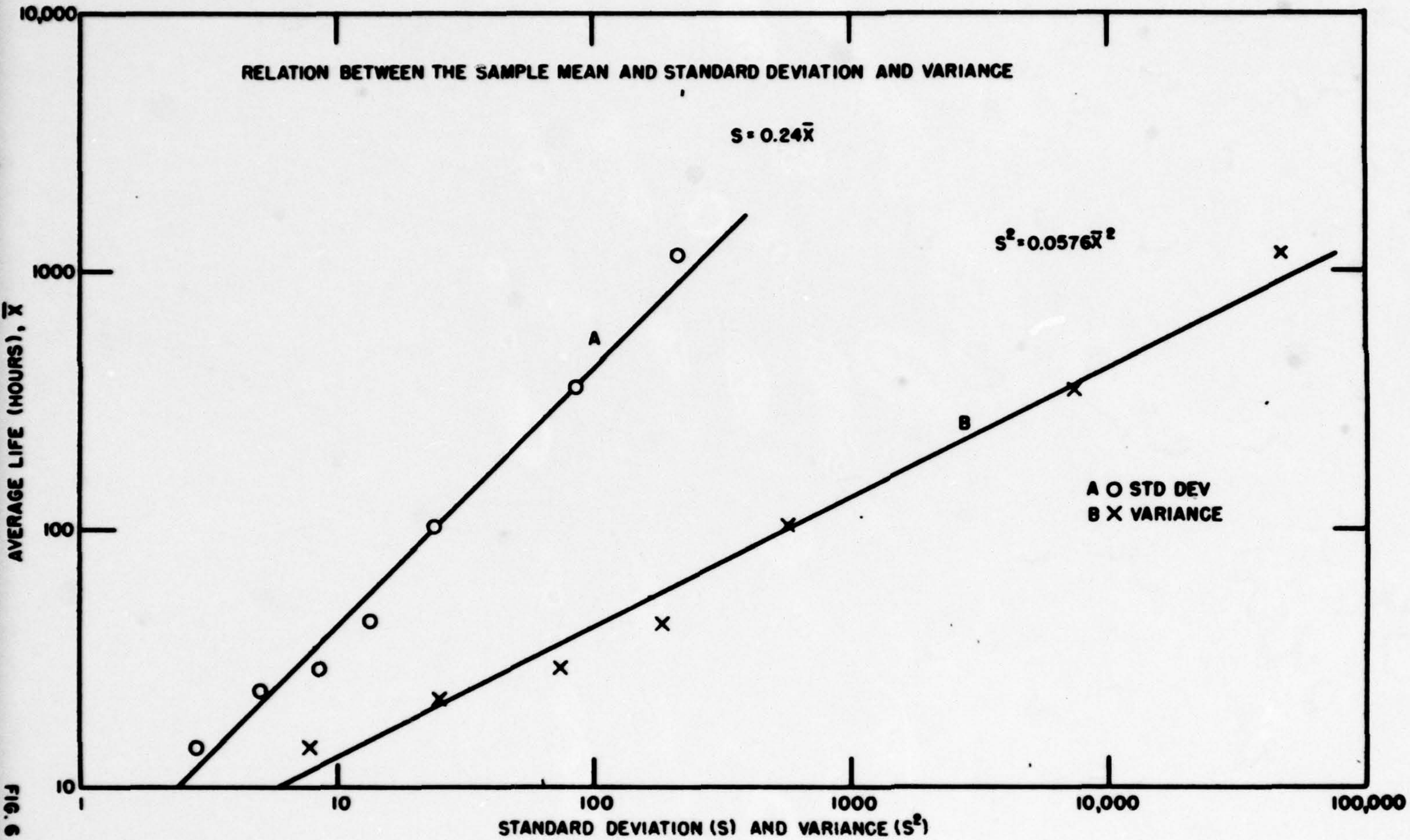
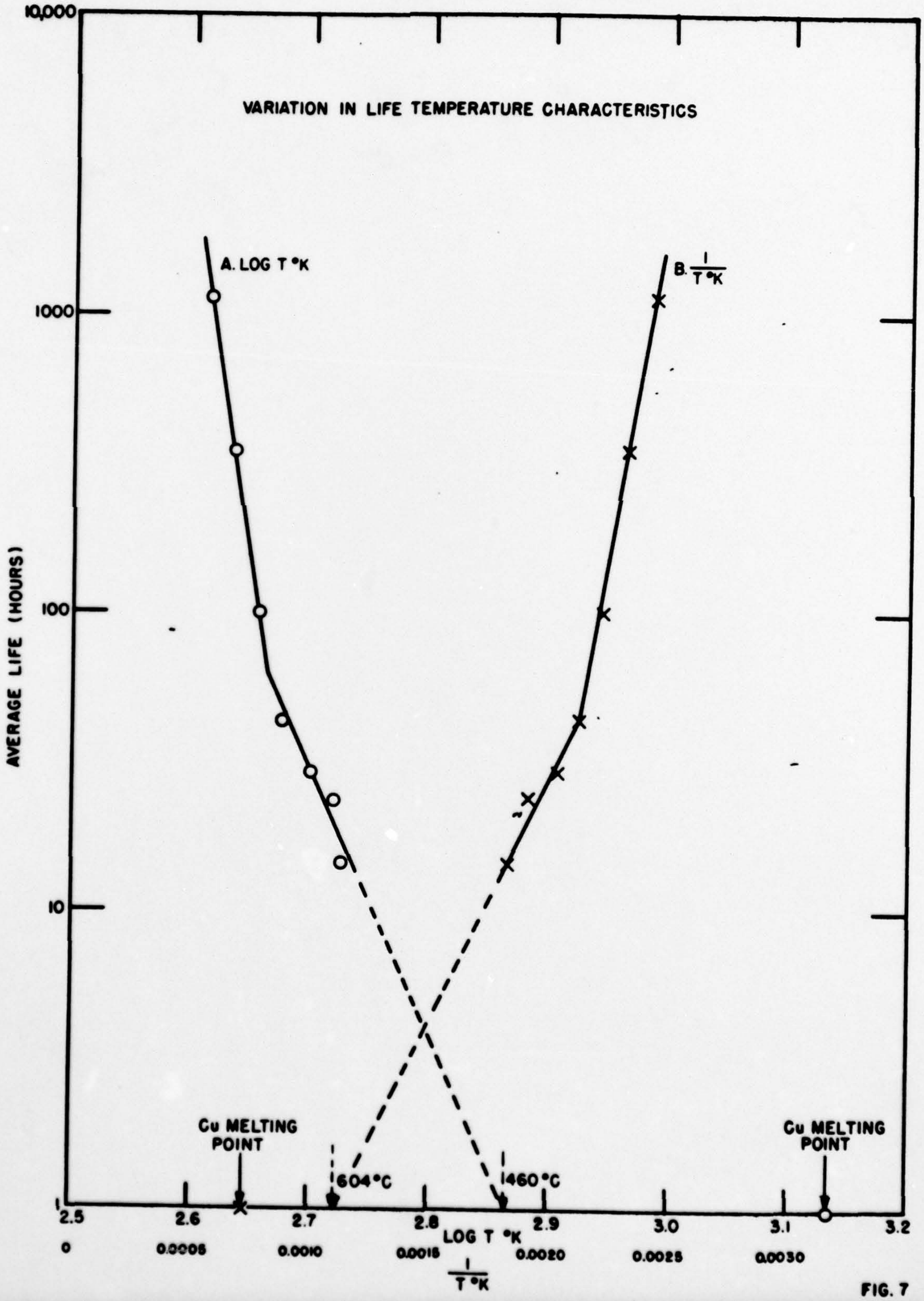

RELATION BETWEEN THE SAMPLE MEAN AND STANDARD DEVIATION AND VARIANCE
S = 0.24X̄
S² = 0.0576X̄²
A
B
A O STD DEV
B X VARIANCE
AVERAGE LIFE (HOURS), X̄
10,000
1000
100
10
1
10
100
1000
10,000
100,000
STANDARD DEVIATION (S) AND VARIANCE (S²)

FIG. 6



VARIATION IN LIFE TEMPERATURE CHARACTERISTICS
AVERAGE LIFE (HOURS)
10,000
1000
100
10
1
A. LOG T °K
B. $\frac{1}{T\,°K}$
Cu MELTING POINT
Cu MELTING POINT
604 °C
1460 °C
2.5
2.6
2.7
2.8
2.9
3.0
3.1
3.2
LOG T °K
0
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030
$\frac{1}{T\,°K}$

FIG. 7

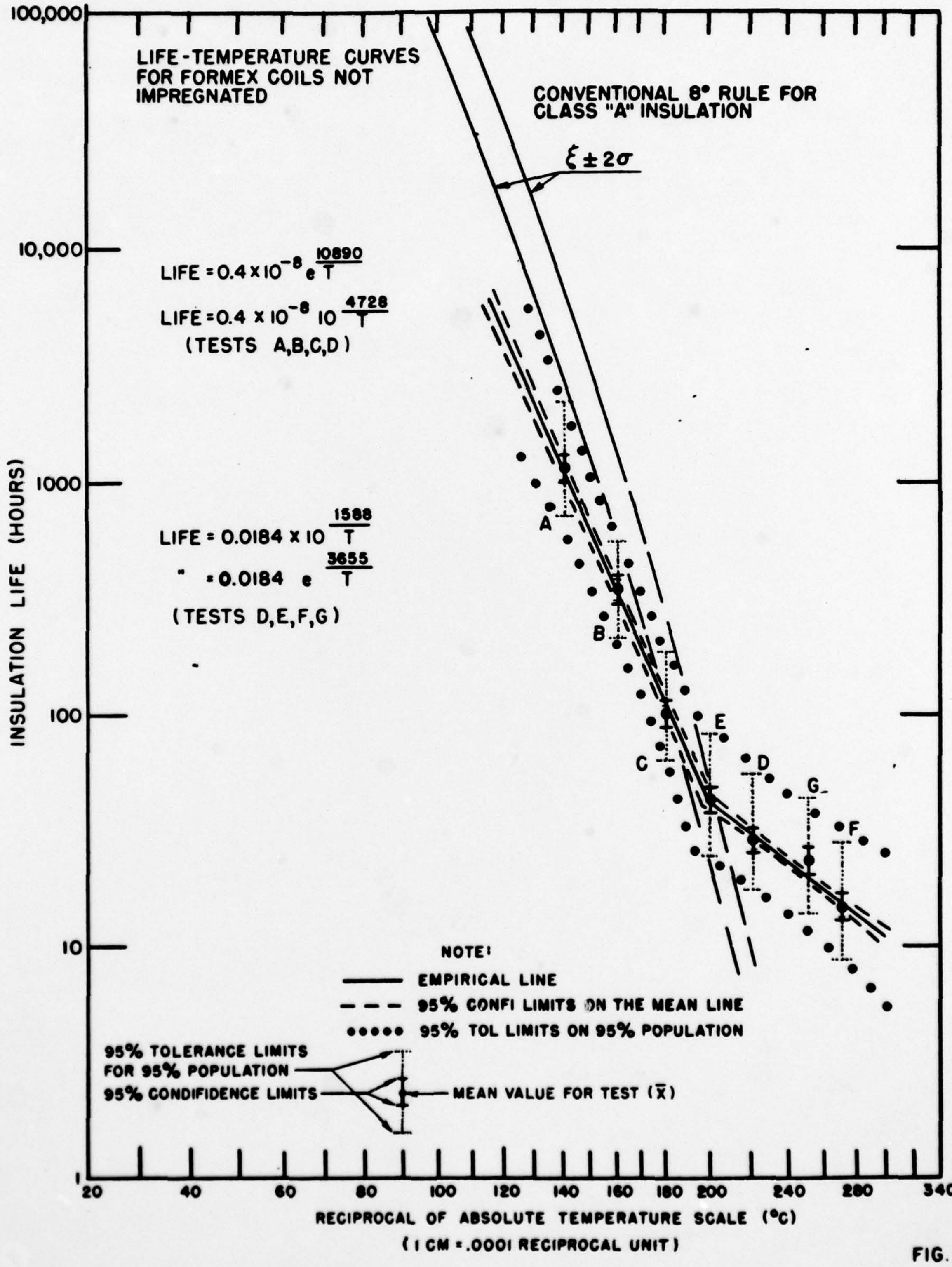

LIFE-TEMPERATURE CURVES FOR FORMEX COILS NOT IMPREGNATED
CONVENTIONAL 8° RULE FOR CLASS "A" INSULATION
ξ ± 2σ
LIFE = 0.4 X 10^-8 e^(10890/T)
LIFE = 0.4 X 10^-8 10^(4728/T)
(TESTS A,B,C,D)
LIFE = 0.0184 X 10^(1588/T)
" = 0.0184 e^(3655/T)
(TESTS D,E,F,G)
A
B
C
D
E
F
G
100,000
10,000
1000
100
10
1
INSULATION LIFE (HOURS)
20
40
60
80
100
120
140
160
180
200
240
280
340
RECIPROCAL OF ABSOLUTE TEMPERATURE SCALE (°C)
(1 CM = .0001 RECIPROCAL UNIT)
NOTE:
EMPIRICAL LINE
95% CONFI LIMITS ON THE MEAN LINE
95% TOL LIMITS ON 95% POPULATION
95% TOLERANCE LIMITS FOR 95% POPULATION
95% CONDIFIDENCE LIMITS
MEAN VALUE FOR TEST (X̄)

FIG. 8

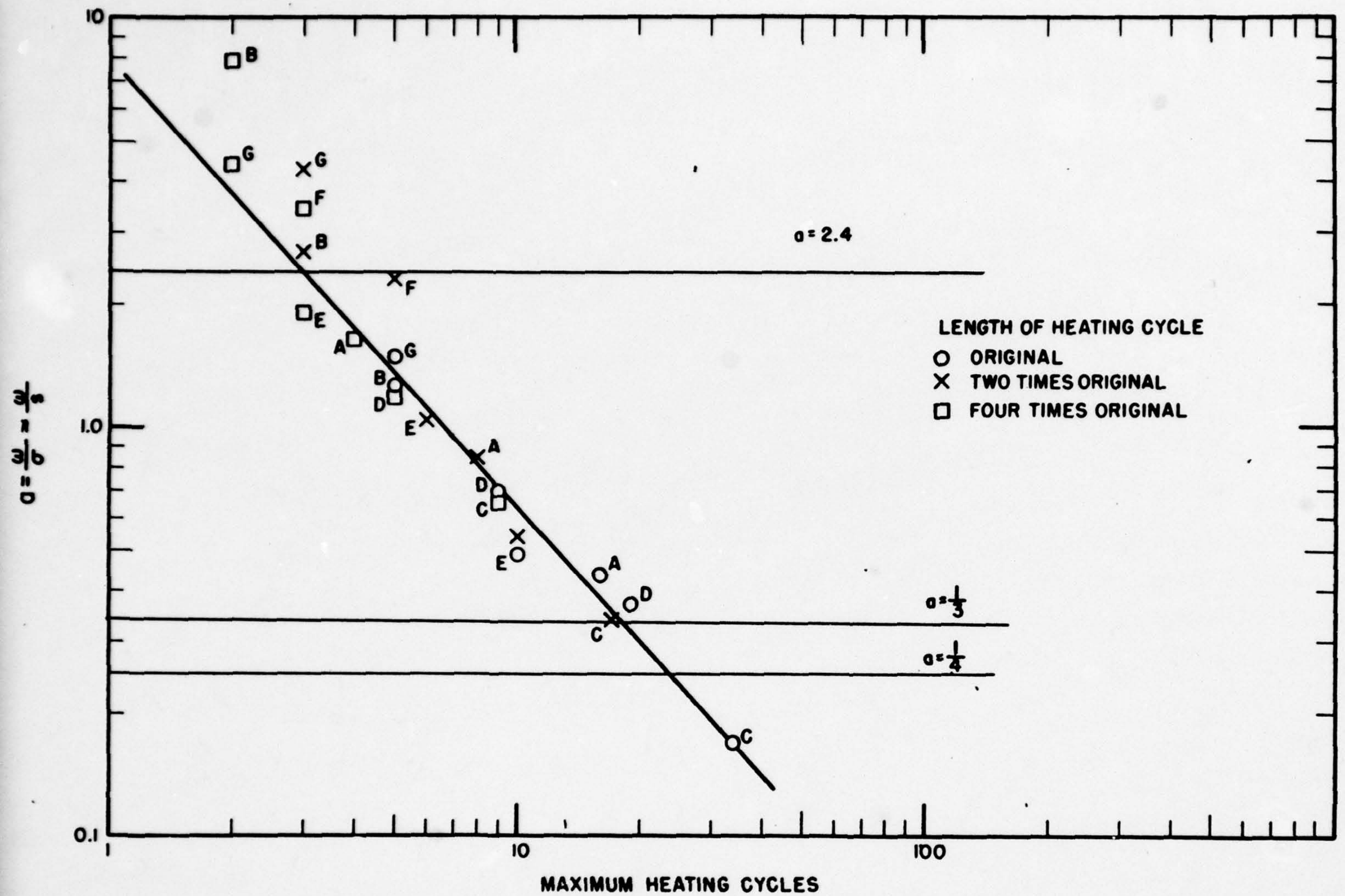


Figure 9. Comparison of the original and reduced experimental data with the proposed limitations on length of heating cycle and maximum number of heating cycles.